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Sidereal Filtering Based on GPS Single Differences for Mitigating Multipath Effects

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ABSTRACT

A new sidereal filtering based on GPS single differences is developed and applied to mitigating GPS multipath effects. Results of simulated and real GPS data show that the proposed method can be used to effectively mitigate the multipath errors based on the reliable single differences converted from double differences. This filtering method is more advantageous in that it is applicable when different satellites are observed on each day and it can be used in real-time.

KEYWORDS: Single differences; sidereal filtering; GPS; multipath effects

1. INTRODUCTION

Multipath effects are one of the most important error sources in high-accuracy global positioning system (GPS) positioning and navigation (Leick, 2004). To separate or mitigate the effects of GPS multipath, a wide variety of approaches have been proposed in the last decades, such as carefully choosing observation sites that do not have potential GPS signal reflectors in their vicinities, using chokering antennas, and improving receiver data processing algorithms. In addition, since the relative geometry of a GPS satellite with respect to an antenna repeats itself approximately every sidereal day (nominally 23 h 56 m 04 s), multipath errors is highly correlated over successive sidereal days, and it is possible to use the "sidereal" satellite repeat period to mitigate this error (Genrich and Bock, 1992; Bock et al., 2000; Nikolaidis et al., 2001). Recent investigations of Choi et al. (2004) showed that correcting coordinates using data from the previous day, shifted by the mean of the individual orbit repeat periods, gave more precise results than using the nominal sidereal period (86,164 s).

However, it is not obvious which time shift to use when different satellites are visible at different times of the day, resulting in the varying mean orbit repeat time. It is therefore preferable if the multipath effects can be removed on a satellite-by-satellite basis. Larson et al. (2007) developed an aspect repeat time adjustment (ARTA) method, using GPS coordinate series to estimate time-varying and site-dependent shifts. However, the limitation of this technique is that it cannot be used in real-time applications such as deformation monitoring.

In this paper, a filtering method, based on satellite-specific single difference observables, is developed for mitigating the multipath effects. First the method of obtaining single differences is briefly described. Then the filtering procedure based on single differences is proposed. Since the proposed method significantly depends on the validity and accuracy of single differences, thus we validate the method with simulated GPS data. Finally, the proposed method is applied to real GPS data and compared with the standard data stacking method. The comparative results and analysis are also presented.

2. OBTAINING SINGLE DIFFERENCES FROM DOUBLE DIFFERENCES

Double differencing is commonly used in high accuracy GPS applications. Let ϕ_A^1 and ϕ_A^2 be observations of satellites 1 and 2 by receiver *A*, and ϕ_B^1 and ϕ_B^2 be observations by receiver *B*. Two single differences can be formed from these four observations,

$$s_{AB}^1 = \phi_A^1 - \phi_B^1 \tag{1}$$

$$s_{AB}^2 = \phi_A^2 - \phi_B^2 \tag{2}$$

A double difference dd_{AB}^{12} can be obtained by differencing the two single differences

$$dd_{AB}^{12} = (\phi_A^1 - \phi_B^1) - (\phi_A^2 - \phi_B^2) = s_{AB}^1 - s_{AB}^2 \tag{3}$$

In order to obtain single differences from double differences, the double difference, *dd*, can be written as the product of a matrix *D* and a vector of single difference, *s*,

$$Ds = dd \tag{4}$$

If there are *n* single differences, then only *n*-1 linearly independent double differences can be formed and the matrix *D* cannot be inverted. However, if an independent constraint on at least one of the single differences is added, as shown in Equation (5), then *D* has a well defined inverse (Alber et al., 2000).

$$\begin{bmatrix} w_1 & w_2 & w_3 \ldots w_n \\ 1 & -1 & 0 \ \ldots \ 0 \\ 1 & 0 & -1 \ \ldots \ 0 \\ & \cdots & \\ 1 & 0 & 0 \ \ldots -1 \end{bmatrix} \begin{bmatrix} s_{AB}^1 \\ s_{AB}^2 \\ s_{AB}^3 \\ \cdots \\ s_{AB}^n \end{bmatrix} = \begin{bmatrix} w_1 s_{AB}^1 + \ldots + w_n s_{AB}^n \\ s_{AB}^1 - s_{AB}^2 \\ s_{AB}^1 - s_{AB}^3 \\ \cdots \\ s_{AB}^1 - s_{AB}^n \end{bmatrix} = \begin{bmatrix} \sum w_i s_{AB}^i \\ dd_{AB}^{12} \\ dd_{AB}^{13} \\ \cdots \\ dd_{AB}^{1n} \end{bmatrix} \tag{5}$$

where $\sum w_i s_{AB}^i$ is the additional constraint and w_i is the satellite-dependent weighting for the site pair *AB*.

For short baselines (e.g. shorter than 1 km), satellite and receiver clock biases are eliminated, and orbital and atmospheric errors are largely cancelled when forming the double-difference observations. However, some other errors, such as multipath, may not be removed with the differencing method. In this paper, we use the post-fit double difference residuals in Equation (5), then setting the sum $\sum w_i s_{AB}^i$ equal to zero produces an inverse where the single differences remain the un-modeled part of the double differences. The un-modeled errors are caused mainly by multipath effects for short baseline applications. Since the amplitude attenuation factor is stronger at low satellite elevation angles due to the gain pattern of a GPS antenna, thus data from low-elevation satellites show a much stronger multipath effects than data from high-elevation satellites (Larson et al., 2007). To improve the constraint, this paper downweights the single differences at low angels by adopting a weighting function $w(\theta)$,

$$w(\theta) = \sin^2(\theta) \tag{6}$$

where θ is the satellite elevation angle.

3 SIDEREAL FILTERING BASED ON SINGLE DIFFERENCES

The implementation of the proposed filtering method includes four main steps.
Step 1: Fix the coordinates of the unknown station and process the data to yield post-fit double-difference carrier-phase residuals for all independent satellite pairs at each observational epoch.

Step 2: Convert double-difference residuals into single-difference residuals epoch by epoch using the method discussed in Section 2.

Step 3: Establish a multipath model by using one day's single-difference residuals. Then the multipath model is shifted and subtracted from single-difference residuals of subsequent days on an epoch-by-epoch and satellite-by-satellite basis. Here shift time of each satellite is determined by sum of orbital repeat periods over the consecutive days.

Step 4: Resolve the final coordinates based on an ambiguity resolution and double-difference residuals reconstructed by the corrected single-difference residuals obtained in Step 3.

A block diagram showing the GPS data filtering procedure is presented in Fig. 1. For convenience of reference, we will name the proposed filtering method based on GPS single differences as single-difference filtering (or SD filtering for brief).

4 SIMULATION STUDIES

The effectiveness of the SD filtering method greatly depends on the validity and accuracy of the converted single differences from double differences. Here we use simulated GPS data to validate the proposed method by comparing the converted single differences with simulated ones.

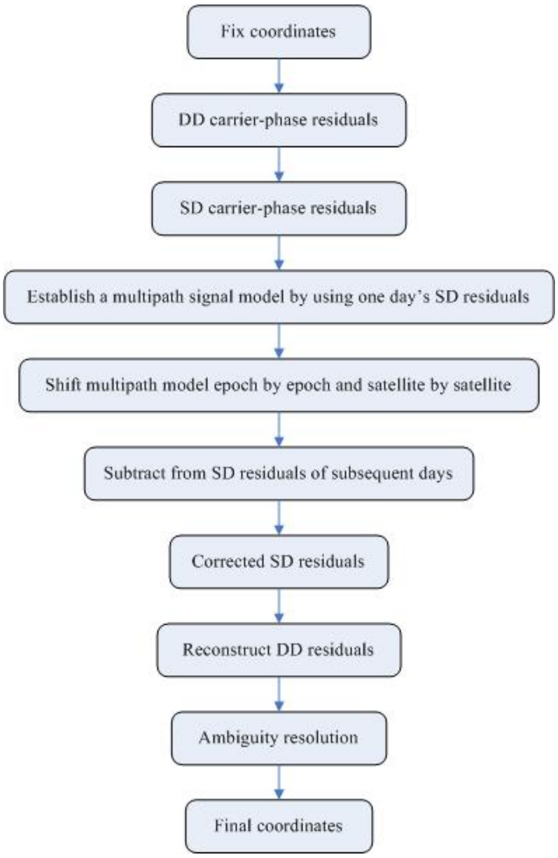

Fig. 1 Sidereal filtering procedure based on GPS single differences (DD: double-difference; SD: single-difference)

4.1 GPS Data Simulator

A GNSS simulator (Satellite Navigation Toolbox 3.0), developed by GPSoft®, is used in this paper to simulate the GPS data. Pseudorange and carrier phase 'measurements' can be generated as true geometric ranges corrupted by many error sources, such as ionospheric and tropospheric refraction and delay, multipath error and measurement noise. Parameters related to simulating the GNSS orbit and various errors are shortly described below.

The 30-satellite GPS constellation (satellite identification number from 1 to 30) is simulated using parameters of perfectly circular Keplerian orbits. The small perturbations associated with the actual satellite orbits are ignored for simulation simplicity. The traditional raised half-cosine profile for zenith delay and elevation angle-dependent oblique factors are used to simulate ionospheric bias. The used model for zenith delay is also called Klobuchar model, assuming that the zenith ionospheric delay can be approximated by half a cosine function of the local time during daytime and by a constant level during nighttime (Klobuchar, 1996). The modified Hopfield model is used to simulate the tropospheric delay, which results in a ranging error of about 3 m for a satellite at the zenith to about 25 m for a satellite at an elevation angle of approximately 5°. Multipath error at zero-elevation angle is modeled by colored or time-correlated noise, which is then scaled by the cosine of the true satellite elevation angle before it is applied to the range measurement. A first-order recursive digital filter having a Butterworth response is used to generate the code multipath error of zero-elevation angle. The carrier-phase multipath error is generated by multiplying the code multipath error by a factor of $(0.05*\lambda)$, where λ is the carrier wavelength in meters. The measurement noise is modeled by a random white noise with normal distribution. In this paper, the standard deviation is 1 meter for pseudorange and $(0.05*\lambda)$ meters for carrier phase.

4.2 Simulation Analysis and Results

GPS data have been simulated for two stations that were about 1.5 km apart from each other over a period of one hour. The satellite elevation cutoff angle was set to 15° and the sampling rate was 1 Hz. Figure 2 illustrates the sky plot of the GPS satellites over the reference station.

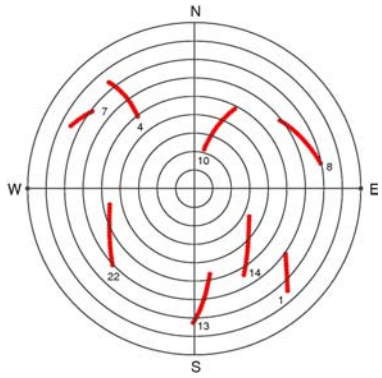

Fig. 2 Sky plot of GPS satellites over the reference station

GPS satellite PRN 10 with the highest elevation angle is selected as the reference satellite when forming the double-differencing observations. Data from three satellites, PRNs 13, 14 and 22, are contaminated by multipath in our analysis. The single-difference carrier-phase residuals are obtained from the double-difference residuals by using the method discussed in Section 2. The converted single-difference residuals compared with the simulated ones (without receiver clock error) for the reference satellite and three multipath-contaminated satellites are shown in Fig. 3. An offset of 5 cm is added to each subplot to separate the time series.

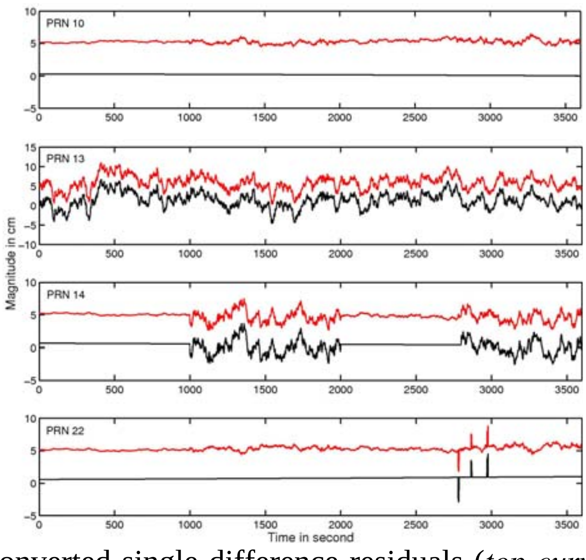

Fig. 3 Comparison of converted single-difference residuals (*top curve in each subplot*) with simulated values (*bottom curve in each subplot*) for reference satellite (PRN 10) and three multipath-contaminated satellites (PRNs 13, 14 and 22)

It is seen from Fig. 3 that the calculated single-difference residuals are quite similar to the simulated ones. The differences between the converted and simulated single-difference residuals are considered caused by the weighting strategy. The root mean square (RMS) values of the differences are about ±0.3 cm for the four satellites, indicating that the weighing function adopted in this chapter (see Equation (6)) works well in all the cases.

5 Experiments with Real GPS Data

GPS observations were collected from two stations that were about 11 m apart from each other, located on the roof of a building in Hong Kong. There are some strong GPS signal reflectors in the vicinity of the stations. A Septentriod PolaRx2@ GPS receiver was used to take observations from 19 (DOY is 323) to 29 November 2005 at a data sampling rate of 1 Hz. A Leica AT504 choke ring antenna was fixed on a concrete pillar as the reference station, while a light weight single-frequency antenna was used for the rover station (see Fig. 4). The satellite elevation cutoff angle was set to 15°.

Fig. 4 Reference and rover stations and site environment

5.1 Resulted Coordinates

The coordinates of the rover antenna were estimated in a post-processing kinematic mode, where the ambiguities were fixed in the processing. Then the resolved coordinates were projected into a map grid system ENU (East, North, up). The original coordinates for a period of about one and a half hour over the 11 consecutive days are used for the analysis and shown in Figs. 5, 6 and 7 for the East, North and up components, respectively. Offsets of 5 cm, 4 cm and 12 cm will be added throughout this section to coordinate series of the East, North and up directions for separating the time series. The mean coordinates have been removed from the results for easy interpretation of the variations.

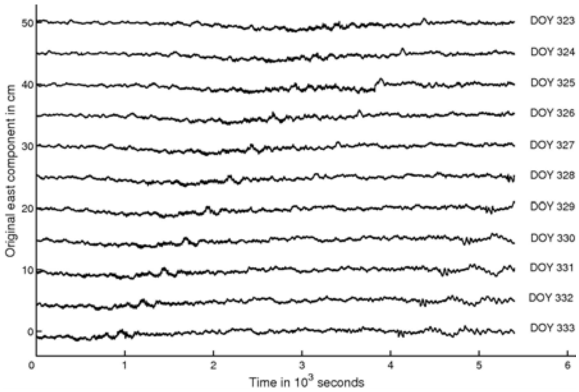

Fig. 5 Original coordinate series from DOYs 323 (*top*) to 333 (*bottom*) in the East direction

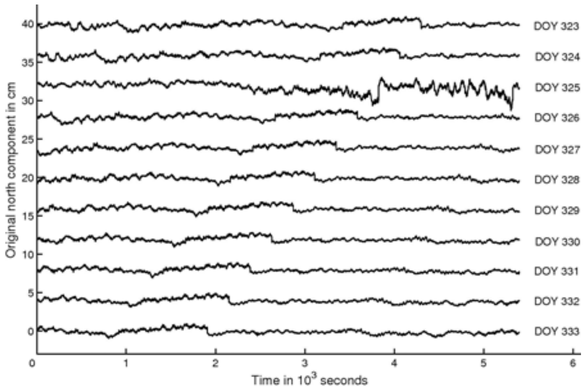

Fig. 6 Original coordinate series from DOYs 323 (*top*) to 333 (*bottom*) in the North direction

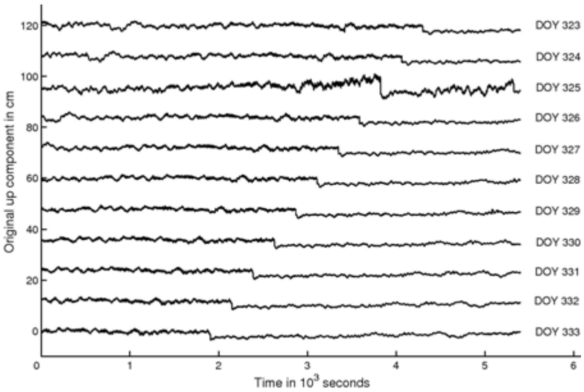

Fig. 7 Original coordinate series from DOYs 323 (*top*) to 333 (*bottom*) in the up direction

To compare the proposed method with the standard data stacking technique (Bock et al., 2000), multipath model for the SD filtering is established by using the single-difference residuals converted from the double-difference residuals on DOY 323; while that for the stacking method is obtained by filtering the coordinate series of DOYs 321 to 323 with the CVVF method (Zheng et al., 2005) and then using moving average technique (Bock et al., 2000). The difference series, obtained by differencing original coordinate series of DOYs 324 to 333 with the multipath models of both methods, are shown in Figs. 8, 9 and 10 for the East, North and up directions, respectively.

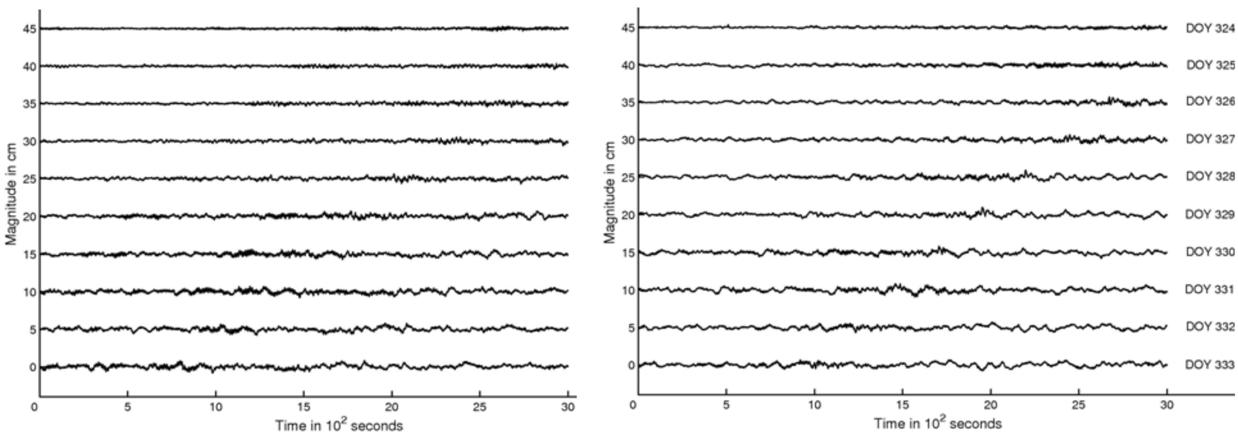

Fig. 8 Difference series after applying the SD filtering method (*left panel*) and the stacking method (*right panel*) from DOYs 324 (*top*) to 333 (*bottom*) for the East direction

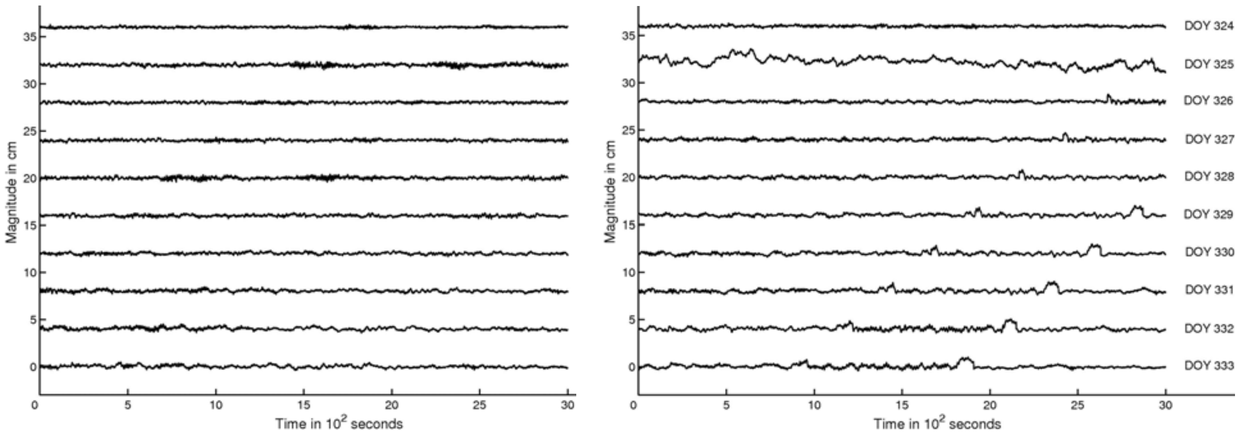

Fig. 9 Same as Fig. 8, except for the North direction

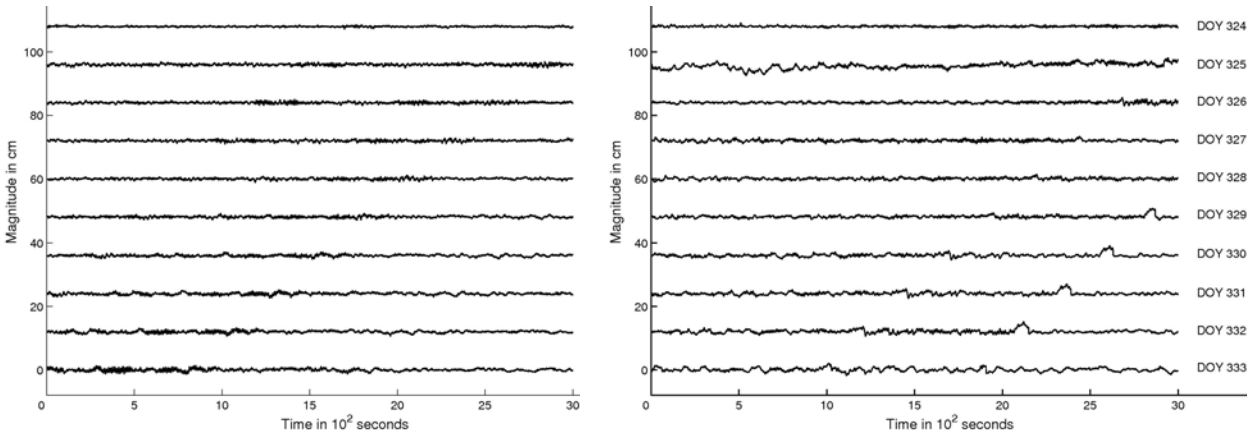

Fig. 10 Same as Fig. 8, except for the up direction

The visual inspection of Figs. 8, 9 and 10 indicates that the difference series after using the SD filtering method gives better results than using the stacking method. Further analysis on the reason of the better performance of the proposed method will be presented in the next section.

5.2 Comparative Analysis

To investigate the effectiveness of the SD filtering and the stacking methods, the percentage of improvements in 3D position accuracy after applying the two methods is listed in Table 1.

DOY	324	325	326	327	328	329	330	331	332	333
SD filtering	75	70	53	50	46	62	67	69	68	65
Stacking	53	-4	35	33	29	21	20	27	29	42

Table 1 3D position accuracy improvements in percentage with the SD filtering and the stacking methods

It is seen from Table 1 that the 3D position accuracy can be improved by about 50-75% with the proposed method. It exhibits the best performance on DOY 325 when compared with the stacking method. As confirmed by NANUs (Notice Advisory to Navstar Users) messages, PRN 6 were maneuvered during the observation period on DOY 325 and thus excluded from coordinate estimate. To get further insight into the filtering property of the proposed method, Fig. 11 shows the number of satellites, vertical dilution of precision (VDOP) values and coordinate series in the up direction for the multipath model and DOY 325. The mean of the satellite numbers and VDOP values on DOYs 321 to 323 is used as the number of satellite and VDOP of the multipath model, respectively. An offset of 8 cm is added to the third subplot in Fig. 11 to separate the up coordinate series for clarity.

a b c

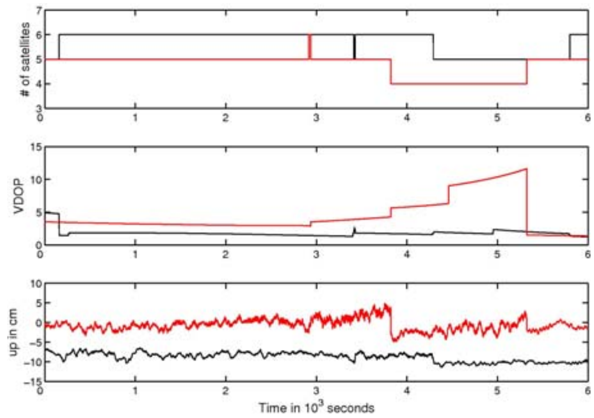

Fig. 11 a Number of satellites for multipath model (*top line*) and DOY 325 (*bottom line*); **b** VDOP values for multipath model (*bottom line*) and DOY 325 (*top line*); and **c** up coordinate components for multipath model (*bottom curve*) and DOY 325 (*top curve*) with offset of 8 cm added

It can be seen from Fig. 11 that compared with the multipath model, fewer satellites on DOY 325 resulted in poorer satellite geometry indicated by the higher VDOP. The highest VDOP values were resulted when only four satellites are visible, corresponding to the large fluctuation of the coordinate series on DOY 325. Therefore, the coordinate series of DOY 325 is quite different from that of the multipath model. It is considered that the coordinate differences caused by missing GPS satellites can degrade the GPS accuracy when the stacking method is applied. The reason for the best performance of the SD filtering method on DOY 325 is due to that this method is on a satellite-by-satellite basis, the missing PRN 6 are thus not included in the final coordinate estimates. This indicates that the SD filtering method is more advantageous than the traditional stacking method in that it can effectively minimize the position errors when different satellites are viewed on each day.

Further analysis shows that although the same satellites were observed on DOYs 324 to 333 (except for DOY 325) during the observation time period, the 3D position accuracy can be improved by about 20-40% with the SD filtering method over the stacking as shown in Table 1. To investigate the reason of the improved performance, Fig. 12 illustrates the comparison of satellite numbers and North coordinate components for the multipath model and DOY 330, and difference series on DOY 330 after applying the stacking and the SD filtering methods. When the stacking method is applied, the optimal shift time is determined by the peak cross-correlation between the multipath model and the coordinate series on subsequent days. An offset of 1 cm is added to the fourth subplot in Fig. 12 to separate the time series for comparison.

It is seen from Fig. 12 that two peaks in the difference series after using the stacking method correspond to two non-zero differences of satellite numbers. It is considered that the left peak is due to that different satellites are used in the position estimates for the multipath model and DOY 330, making some of the coordinates not exactly repeatable; while the right peak is caused by some of the GPS satellites not shifted by its optimal shift time. Since multiple satellites contribute to each coordinate, it is considered that the stacking method necessarily forces a compromise among the satellite-specific optimal time shifts. Compared to the SD filtering method, both peaks are removed from its difference series, indicating that the proposed method can not only ensure the same satellites in position estimates, but also provide more precise results than the stacking method by shifting each satellite by its individual shift time instead of a single time shift.

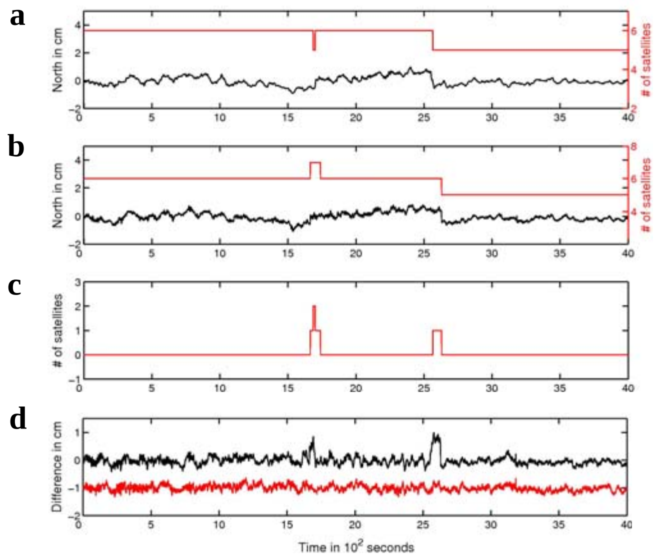

Fig. 12 a Number of satellite (*line*) and North coordinate component (*curve*) for multipath model; **b** number of satellite (*line*) and North coordinate components (*curve*) on DOY 330; **c** difference of satellite numbers between multipath model and DOY 330; and **d** difference series on DOY 330 after using the stacking (*top curve*) and the SD filtering (*bottom curve*) methods with offset of 1 cm added

6 CONCLUSIONS

A filtering method based on GPS single difference observations has been proposed for mitigating GPS multipath effects. Test results have shown that the accuracy of GPS measurements can be improved by about 50-75% with the proposed method. Tests have also shown that about 20-40% improvements in GPS accuracy can be achieved with the proposed method when compared with the standard data stacking method. The proposed method is more advantageous in that it is applicable when different satellites are observed on each day. It can not only exclude bad satellites from final position estimates, but also ensure the same satellites used for multipath model and subsequent coordinate series. The proposed method is more practical in that it can be implemented in real-time.

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