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Carrier phase analysis to mitigate multipath effect

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ABSTRACT

Although researchers have devised different methods to detect and correct effects of multipath, it remains one of the challenges in navigation receiver design. This paper explores the phase variations of the incoming carrier in the presence of multipath in GPS signals. Due to this change, while tracking an incoming signal, energy is shifted from the I channel to the Q channel. A novel technique has been presented to analyse the pattern of this energy shift in an ideal scenario to detect and accurately estimate the amplitude, delay and phase of the reflected signal. A perfect triangle has been used to depict ideal autocorrelation peak of a GPS signal. The algorithm is then applied to simulated GPS signals and it has been shown that errors are introduced in multipath estimates because GPS autocorrelation is not a perfect triangle. Tracking output from a software receiver is then used to detect and estimate multipath. A novel variable called Early late phase (ELP) has been proposed for multipath estimation. It has been shown that ELP performs better when carrier phase difference between line of sight and reflected signal is $\pi/2$ or its multiples. It gives worst performance when this difference is π or its multiples, because in this case there is no energy shift to Q-channel as the reflected signal is also matched to line of sight signal. However, this problem may be solved if the same technique is used for both L1 and L2C to aid each other.

This paper provides background, explanation and initial results for the proposed algorithm. There are still few limitations in the algorithm, which needs to be addressed by more extensive experiments and analysis.

KEYWORDS: Multipath, GPS, carrier phase

1. INTRODUCTION

Since the commissioning of first navigation satellite in 1977, researchers had been trying to increase the positioning accuracy by developing algorithms for reducing the effect of different interferences (Hannah, 2001). Although there has been a significant increase in accuracy since then, there is still motivation for even higher accuracy due to developing of new applications requiring precise positioning. GPS, being a CDMA based system is severely affected by the reflections of transmitted signal, known as multipath effect. Initially GPS was proposed for aircraft, ship and vehicle navigation. In such situations, receiving a reflected signal is much less probable because of absence of surroundings from where the signal can reflect and ground reflections are easily blocked by the antenna. However, with applications being proposed to use GPS in urban canyons and even indoors, a lot of research in past few years has been focused to reduce the effect of multipath on navigation solution. As a result different algorithms have been proposed using signal processing techniques in the software. Some of the widely used techniques are Narrow correlators (Dierendonck *et al.*, 1992), Multipath elimination technique (MET) (Townsend and Fenton, 1994), Multipath estimating delay locked loop (MEDLL) (Townsend *et al.*, 2000). These techniques and most of others use sum of squares of in-phase (I) and quadrature phase (Q) outputs of the correlator. However, the multipath component is different from the line of sight (LOS) of same signal in terms of code delay and carrier phase, and by using I^2+Q^2 although the code phase difference is exploited but carrier phase difference is not. This provided motivation to analyse the carrier phase to detect and estimate the multipath effect. Once the estimates are computed, multipath can be removed from the incoming signal and the line of sight signal can thus be tracked.

This paper presents an algorithm which analyses the energy variation in I and Q channel outputs to exploit the difference of carrier phase between LOS and reflected signals from the GPS satellites. The algorithm was implemented in Matlab and tested on simulated signals in various scenarios developed. The initial results are presented in this paper, which are quite encouraging and provide motivation for extensive experiments and further research towards refinements to the algorithm in future.

2. MULTIPATH EFFECT ON PERFECT TRIANGLE

2.1 Why Triangle?

The basic concept behind the development of this algorithm is explained using a simple triangle, an idealized form of an autocorrelation triangle in GPS satellites. Consider a triangle function $v(t)$ shown in Figure 1 to be the ideal autocorrelation result of incoming signal and local code. Although in reality the autocorrelation result has noise which means that the triangle is not completely perfect and the regions outside the triangle are not exactly zero, however, those noise effects are not considered at the moment to demonstrate the usefulness of carrier phase analysis to detect and estimate multipath. The real part of $v(t)$ corresponds to I channel of the correlator output, while imaginary part of it corresponds to Q channel. In ideal scenario considered here, the Q channel is zero.

2.2 Multipath effect

The autocorrelation result of a reflected signal received at a GPS receiver is different from that of LOS because the two signals have different attenuation, time delay and carrier phase. The autocorrelation function $r(t)$ of the reflected signal can be found using $v(t)$ as given in equation 1.

$$r(t) = \alpha e^{j\Phi} v(t - \tau) \quad (1)$$

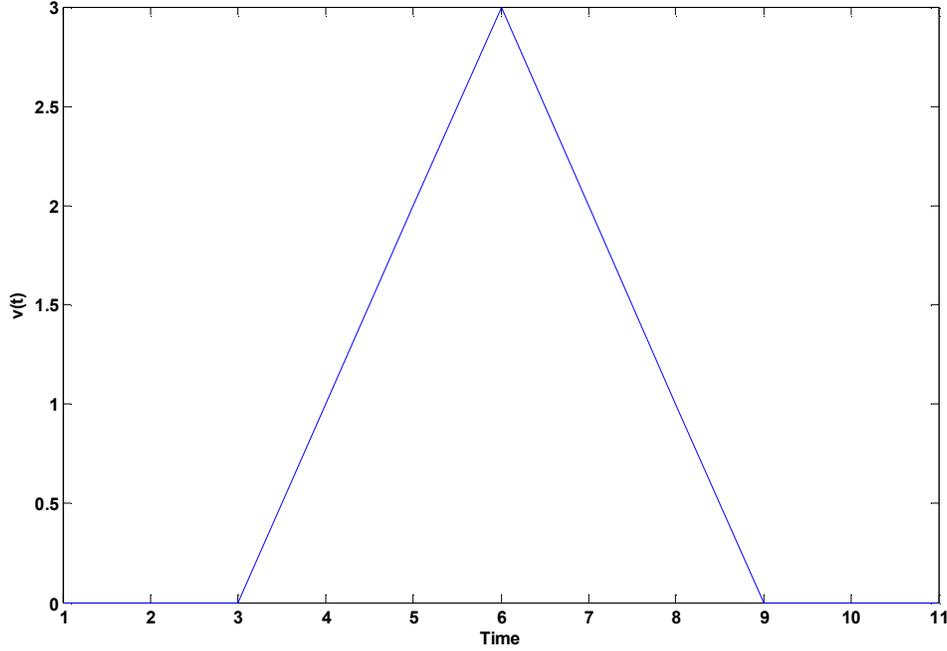


Figure 1. Perfect triangle – Ideal autocorrelation result

At the receiver both reflected and LOS signals are received, which can be given by equation 2 and 3.

$$g(t) = v(t) + r(t) \quad (2)$$

$$g(t) = v(t) + \alpha e^{j\Phi} v(t - \tau) \quad (3)$$

where α is the attenuation of the reflected signal as compared to LOS signal which is assumed to be received without attenuation, Φ is the phase difference and τ is the time delay between the LOS and the reflected signal. The absolute values of $v(t)$, $r(t)$ and $g(t)$ are plotted in Figure 2. The absolute value corresponds to I^2+Q^2 operation in GPS receivers. It can be shown that it is difficult to estimate or even detect multipath from this plot.

As can be seen from equations 1-3, $v(t)$ was completely real but $g(t)$ also has imaginary component. The real component corresponds to a component of received signal which is in-phase with LOS and imaginary one corresponds to the component of received signal which is out of phase with LOS. The amount of energy split in these two components depends on the angle Φ . In other words, the analysis of energies in real and imaginary components of $g(t)$ can

help to estimate $r(t)$. In terms of GPS terminology, it implies that individual analysis of I and Q channel can be useful in multipath detection and elimination.

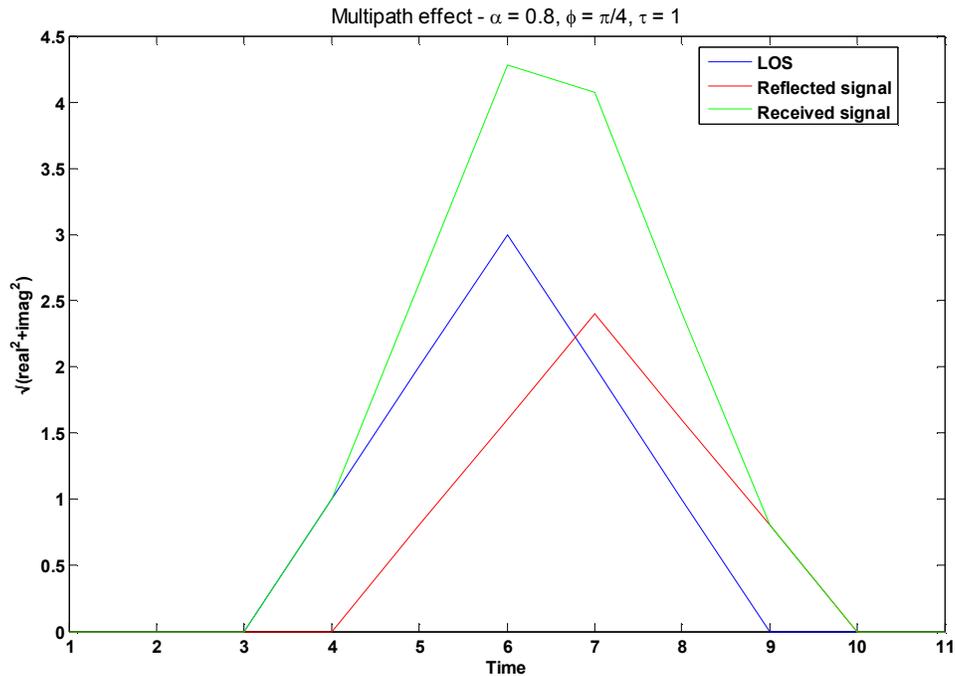


Figure 2. Perfect triangle – in presence of multipath

2.3 Multipath estimation

As mentioned above, its rather difficult to estimate multipath effect using sum of squares of real and imaginary components of $g(t)$. Thus, a plot of real versus imaginary component has been used for multipath detection and estimation. As opposed to Figure 2, the plot in Figure 3 shows the shift in energy from real to imaginary components, which can be used for multipath estimation. The presence of any energy in imaginary component of $g(t)$ shows the presence of multipath. Using this criterion alone, multipath can be detected in almost all the scenarios. The only exception to this is, when the $\Phi = 0$ or $\Phi = \pi$. Infact the case of $\Phi = \pi$ can also be detected as the real component is negative in this case, which can only happen in presence of multipath.

Apart from detection, this plot can also be used to estimate the multipath. The last leg of plot before going back to origin corresponds to the autocorrelation peak because of reflected signal or $r(t)$ only because the autocorrelation peak of LOS has gone to zero at this point. Thus the angle at which it goes back to origin gives the phase of reflected signal or in other words Φ can be found by calculating this angle. Figure 3 shows the plots for different values of Φ while keeping attenuation and delay constant. It can be seen that Φ can be found using above method in all the cases.

Figure 4 shows the similar plots for varying delay between the LOS and the reflected signal. This delay can only be positive as reflected signal always reach receiver after LOS, thus negative values are not considered. It can be seen that the point where imaginary component of $g(t)$ starts to have nonzero value, corresponds to the value of real component of $g(t)$ at time

when autocorrelation peak of reflected signal gets nonzero value. Thus the time difference between the first non-zero value of $g(t)$ and the first nonzero imaginary component of $g(t)$ is equal to τ . Mathematically, it can be found using equation 4.

$$\tau = \frac{g(t_i)}{\lambda} \quad (3)$$

where t_i is the time when $g(t)$ got first nonzero imaginary value and λ is the slope of perfect triangle $v(t)$. In the example considered here, λ is taken to be equal to 1 for simplicity as can be seen from Figure 1. Therefore, the time delay between LOS and reflected signal is equal to $g(t_i)$. Figure 4 shows that the time delay has been estimated accurately for all the three cases tested using equation 3.

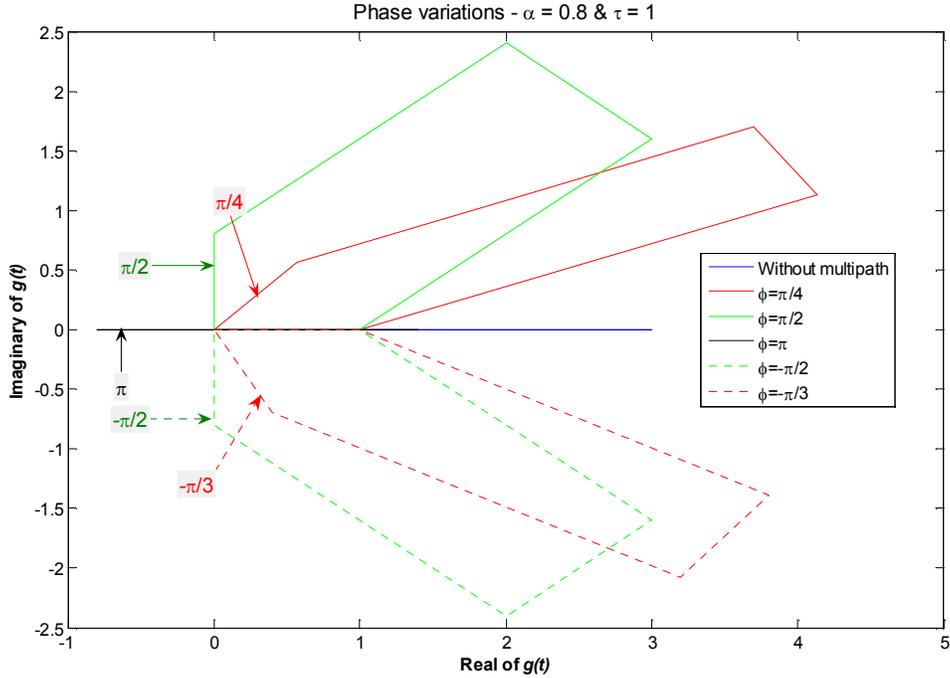


Figure 3. Effect of multipath – phase variations while keeping attenuation and delay constant

Figure 5 shows the effect of attenuation of reflected signal, while keeping time delay and phase constant. The area of the polygon formed while plotting imaginary component of $g(t)$ against its real one, has been used for calculating the reflected signal attenuation. It has been found that for a given delay and phase difference between LOS and reflected signal, the area of this polygon is directly proportional to the attenuation of the reflected signal, even when reflected signal is stronger than LOS. Thus, if delay and phase difference can be found as described above, the attenuation can be estimated using the area of the polygon.

2.4 Algorithm

On the basis of analysis presented above, an algorithm has been developed to accurately detect and estimate the multipath effect in a received signal. Figure 6 shows the block diagram of the proposed algorithm. It first computes the imaginary component of $g(t)$. If it is

equal to zero, then there is no multipath present. Otherwise, the algorithm will first compute Φ using the angle at which imaginary component of $g(t)$ goes back to zero and τ using the first nonzero value of this component. Once they are calculated, the area of polygon is computed for reflected signal as strong as LOS, or in other words for attenuation of 1. The area of polygon obtained from the incoming $g(t)$ is then computed and divided with one obtained earlier. As the area of polygon is directly proportional to the attenuation of reflected signal, the resultant would give attenuation of the reflected signal.

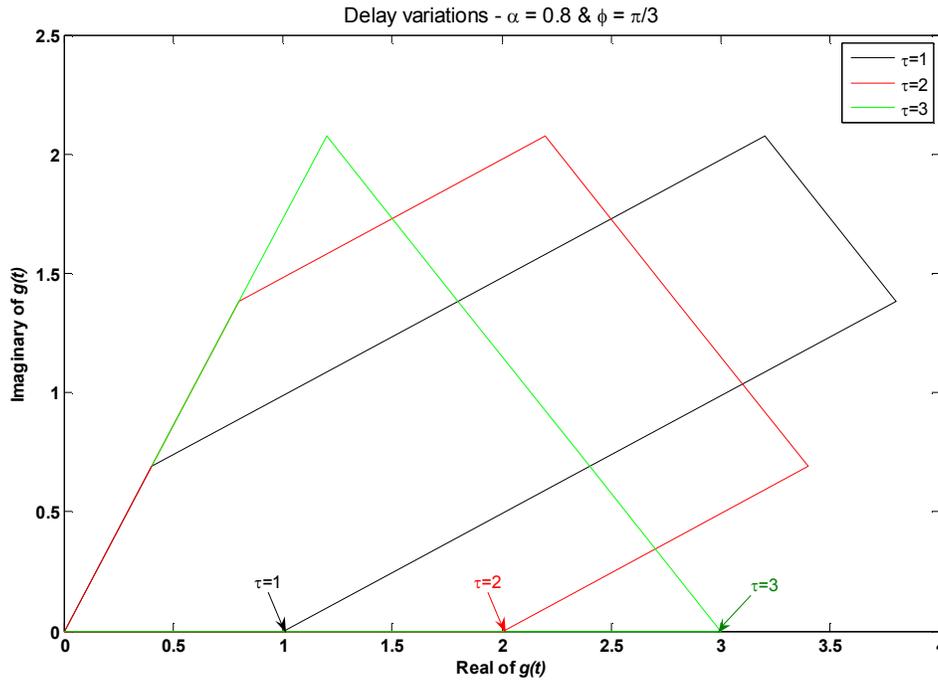


Figure 4. Effect of multipath – delay variations while keeping attenuation and phase constant

3. MULTIPATH EFFECT ON GPS SIGNAL

The analysis and algorithm presented in previous section is true for a perfect triangle, however in reality the autocorrelation of a GPS signal is noisy and does not give a perfect triangle. As mentioned above, the real and imaginary components of $g(t)$ corresponds to the outputs of I and Q channels of correlator. Thus, in this section I-channel output is plotted against that of Q-channel for 1 millisecond of correlation result. This plot is then used to detect and estimate the multipath. As the phase difference and time delay between LOS and reflected signal are dependent on each other in real GPS signal, only phase difference would be estimated because it is less affected by the noise as compared to time delay.

The experiments were setup to analyse the effect of multipath on I and Q channel outputs. The satellite signals with random noise were generated in Matlab using sampling frequency of 5.714 MHz and carrier frequency of 1.405 MHz. The generated received signal was band pass filtered by 2 MHz filter before correlation processing. The experiments were run for acquiring satellite with PRN 7 in presence of 6 other satellites having slightly different carrier frequency due to relative Doppler shift.

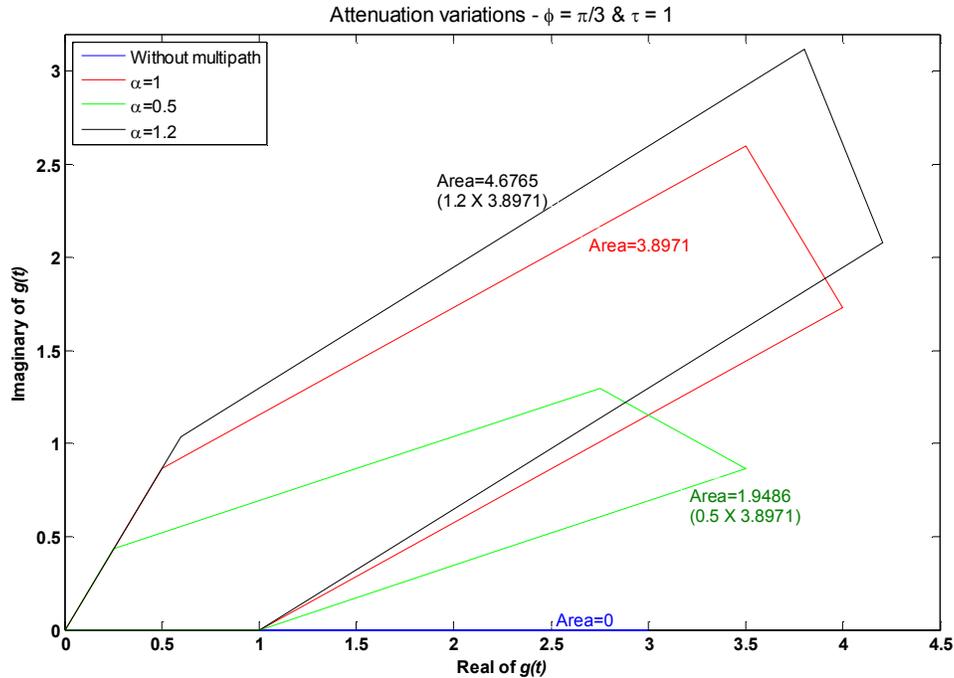


Figure 5. Effect of multipath – attenuation variations while keeping phase and delay constant

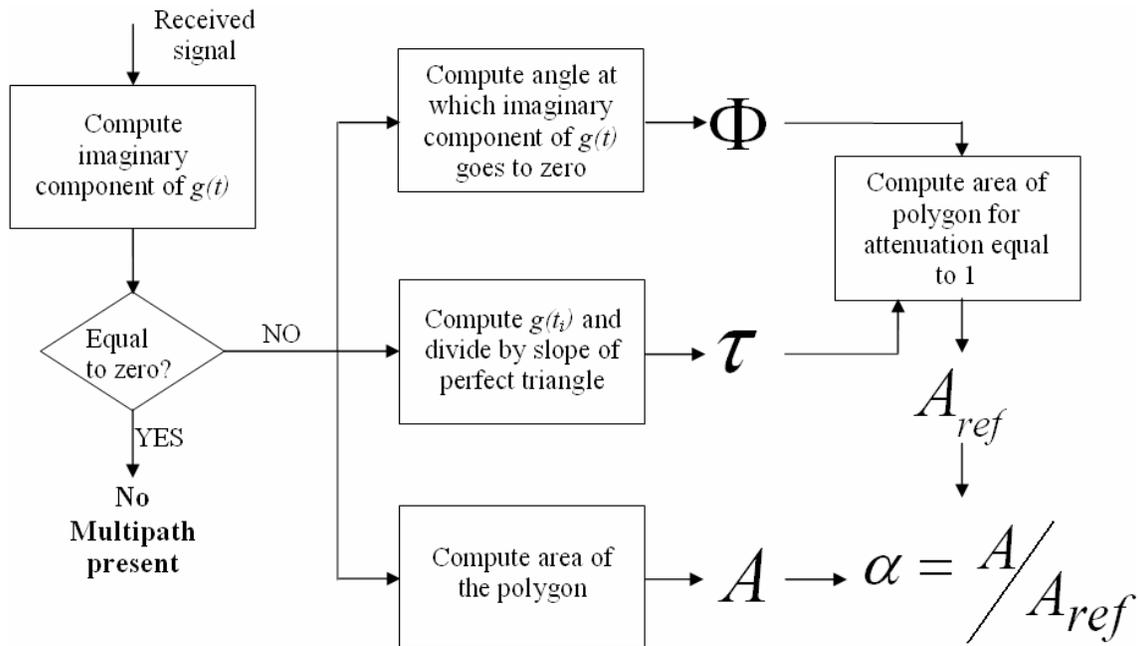


Figure 6. Algorithm to detect and estimate multipath for perfect triangle

Firstly it was assumed that the local carrier phase is matched with LOS carrier. Figure 7 shows the results of this experiment for different multipaths. It can be seen that there is a significant energy in Q-channel for the phase difference of around $\pm\pi/2$, which is the phase difference between local carriers of I and Q channel. This means that it is much easier to detect multipath around that phase. Detection may also be possible for a phase difference of π as there is one positive and one negative peak in I-channel in that case. When the phase

difference is around 2π or its multiples, neither the I nor Q channel get any extra information, and thus the proposed method may not work in such cases.

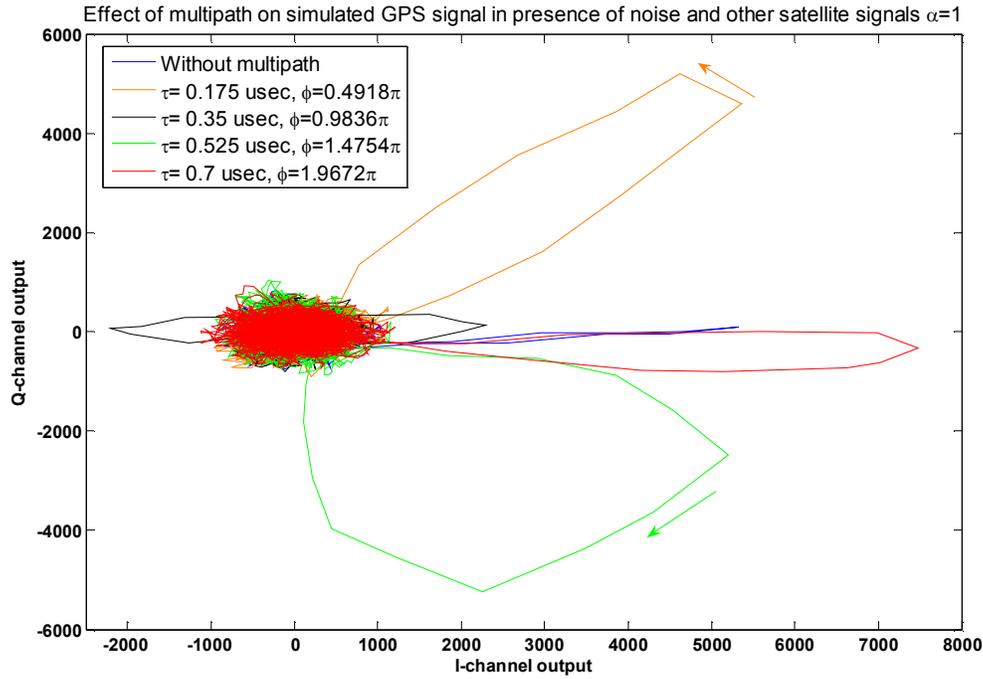


Figure 7. PRN 7 in presence of PRN 12 (500 Hz), 15 (200 Hz), 19 (2000 Hz), 25 (3000 Hz), 28 (-2000 Hz), 31 (-3000 Hz) and 18 dB stronger noise – local carrier phase matched with PRN 7

Once detected, the area and shape of polygon formed in I versus Q channel output plot can help estimate the attenuation and phase of the reflected signal. Attenuation can be found using the area as was done in case of perfect triangle, however due to noise the area would not be exactly proportional to the attenuation, thus results would not be that accurate. Figure 8 shows the error in attenuation calculation for simulated signal, using area at $\alpha=1$ as reference. It can be seen that the effect of noise is more pronounced at higher attenuations (lower values of α). Moreover, error in attenuation calculation for $\tau=0.175 \mu\text{sec}$ is more than that for $\tau=0.525 \mu\text{sec}$. It is because of the reason that for given attenuation, area of polygon for $\tau=0.525 \mu\text{sec}$ is larger than for $\tau=0.175 \mu\text{sec}$ and thus the same amount of noise has lesser impact.

The phase difference can be estimated by using the location of polygon. If it is located in first or second quadrant, then it is more likely that the phase difference would be around $\pi/2$ and if it is located in third or fourth quadrant, then it should be around $3\pi/2$. However, this is only true when local carrier is exactly matched with carrier of LOS signal. This is not true in presence of multipath, as in that case the phase of the local carrier would be pushed by tracking loop to maximize I^2+Q^2 .

The effect of carrier phase mismatch can be seen from Figure 9, which shows same scenario as Figure 7 but with carrier phase shifted by $\pi/2$. It can be seen that whole plot has been rotated by $\pi/2$. The phase of local carrier in the presence of multipath would be between the phase of LOS and reflected signals or in other words all these polygons would be centred on x-axis in any case. However, a closer look at Figure 7 and Figure 9 would reveal that the polygon of phase difference of around $\pi/2$ and $3\pi/2$ are still distinguishable even when local carrier has phase mismatch with LOS signal. In both figures the polygon generated by $\Phi \approx \pi/2$

moves in a clockwise way to return to zero, or in other words if carrier tracking loop is locked in the middle of the polygon, the early correlator output would always be on its right side and late on at its left. Similarly, polygon generated by $\Phi \approx 3\pi/2$ moves in anticlockwise direction to return and would have early correlator output on left and late one on its right. This difference would be exploited in next section, when the tracking results from a software receiver would be used to estimate multipath.

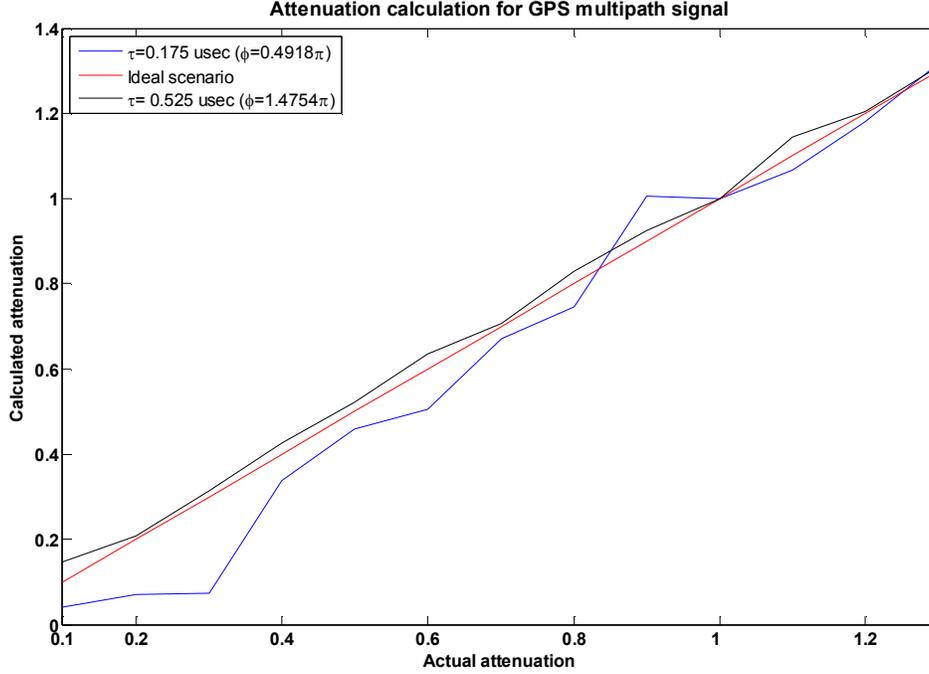


Figure 8. PRN 7 in presence of PRN 12 (500 Hz), 15 (200 Hz), 19 (2000 Hz), 25 (3000 Hz), 28 (-2000 Hz), 31 (-3000 Hz) and 18 dB stronger noise – Error in attenuation calculation using proposed method on simulated GPS signals

4. MULTIPATH ESTIMATION FOR SIMULATED GPS SIGNAL USING SOFTWARE RECEIVER

4.1 Early and late phase analysis

A software receiver developed in Matlab (SoftGPS Project, 2006) has been used to obtain tracking results for different multipath scenarios. The software uses discriminator given by equation 4, whose output gives the phase error.

$$E(t) = \frac{\tan^{-1}\left(\frac{Q_P(t)}{I_P(t)}\right)}{2\pi} \quad (4)$$

where Q_P and I_P are the I and Q channel prompt outputs of the correlator. The carrier loop would always keep $E(t)$ close to zero or the phase locked with the prompt signal. Thus, it is expected that phase difference of Early and Late outputs of the correlator would change in the presence of the multipath, as they would be sitting on the shoulders of polygon discussed above.

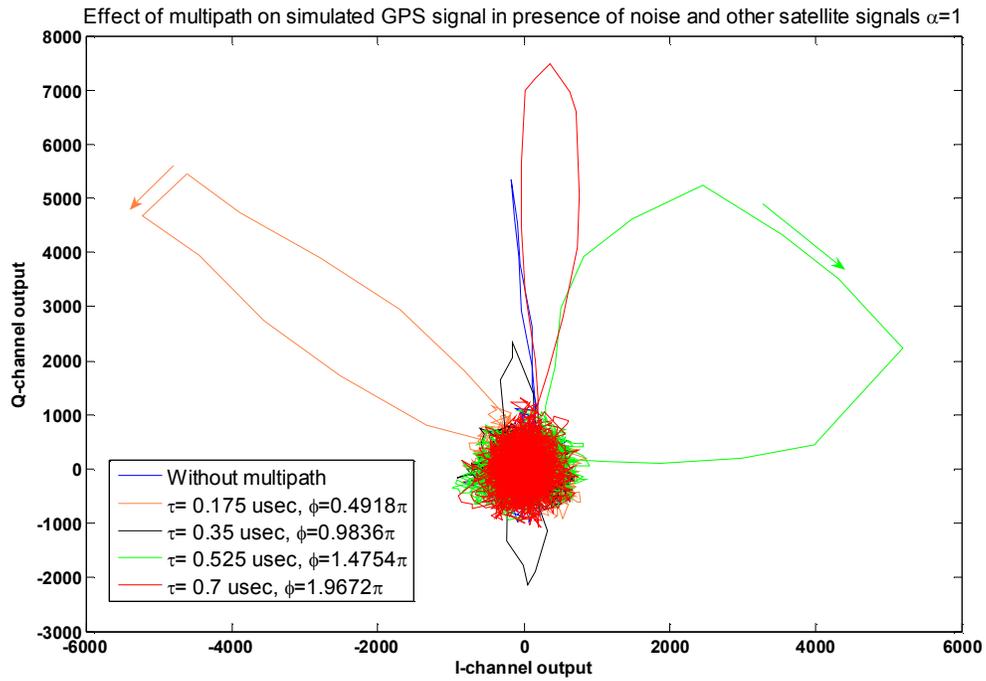


Figure 9. PRN 7 in presence of PRN 12 (500 Hz), 15 (200 Hz), 19 (2000 Hz), 25 (3000 Hz), 28 (-2000 Hz), 31 (-3000 Hz) and 18 dB stronger noise – local and PRN 7 carrier phase difference of $\pi/2$

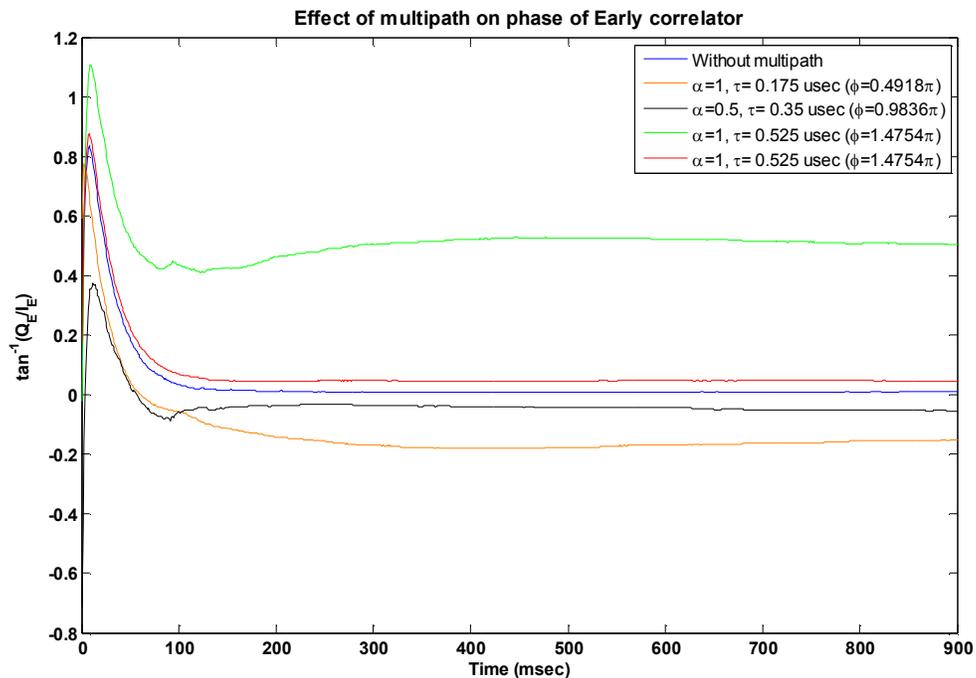


Figure 10. Multipath effect on phase of early correlator output for PRN 7 in noiseless environment

Figure 10 and Figure 11 shows the effect of multipath on phase of early and late correlators for 900 milliseconds computed after every 1 millisecond in noiseless conditions. First 100

milliseconds can be ignored as the loop is settling down in that period. Once stabilized, it can be seen that for $\Phi \approx \pm\pi/2$ the phase of both early and late is significantly different from the case when there is no multipath. As expected, the early phase for $\Phi \approx \pi/2$ is positive and late phase is negative, while that for $\Phi \approx 3\pi/2$ it is other way round. However as mentioned before, the detection of multipath for $\Phi \approx \pi$ or $\Phi \approx 2\pi$ is relatively tougher as the early and late phases in those cases are quite close to without multipath case.

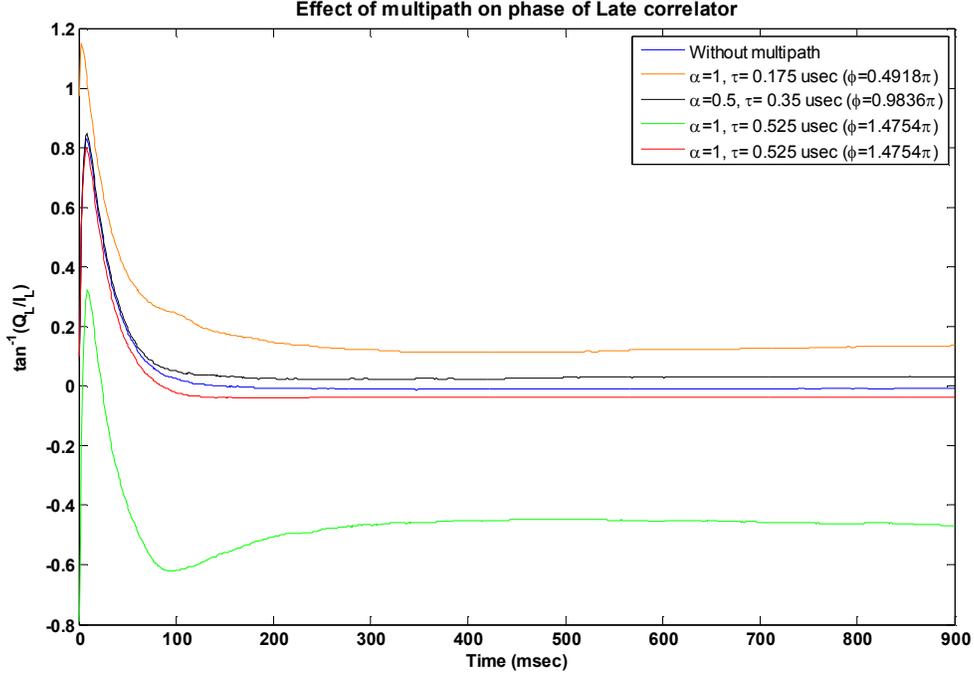


Figure 11. Multipath effect on phase of late correlator output for PRN 7 in noiseless environment

Although the multipath detection seems relatively easier in noiseless conditions by either using early or late correlator output, it is much tougher in real time situations. Figure 12 shows the early correlator output for $\Phi \approx \pi/2$ and $\Phi \approx 3\pi/2$ in presence of 18 dB stronger noise and 7 other satellite signals, to depict real time conditions. The presence of multipath was clearly detectable for these two values of Φ in noiseless environment; however it is not that clear in real time noisy scenario. In order to remove this noisy effect, the phase is smoothed by averaging over 50 milliseconds. The resultant plots are given in Figure 13, which clearly shows that smoothing has made multipath detection easier.

4.2 Early and late phase (ELP) variable

So far only one of early or late correlator output has been looked at in a single plot. A single novel variable, early late phase (ELP) has been proposed to exploit both of the correlator outputs. In Figures 10-13, phase difference between prompt and either of early or late has been considered. ELP is the phase difference between early and late, which would exploit the both of those phase differences. Mathematically, ELP is given by equation 5.

$$ELP(t) = \tan^{-1} \left(\frac{Q_L(t)}{I_L(t)} - \frac{Q_E(t)}{I_E(t)} \right) \quad (5)$$

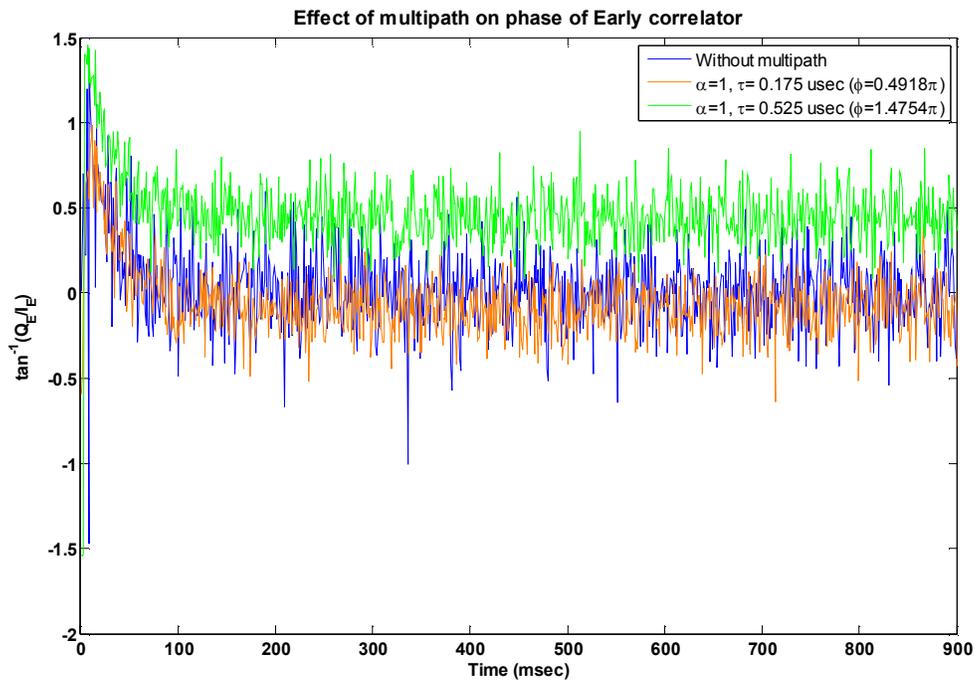


Figure 12. Multipath effect on phase of early correlator output for PRN 7 in presence of 18 dB noise and 7 other equally strong satellite signals (PRN: 3, 12, 15, 19, 23, 28 & 31)

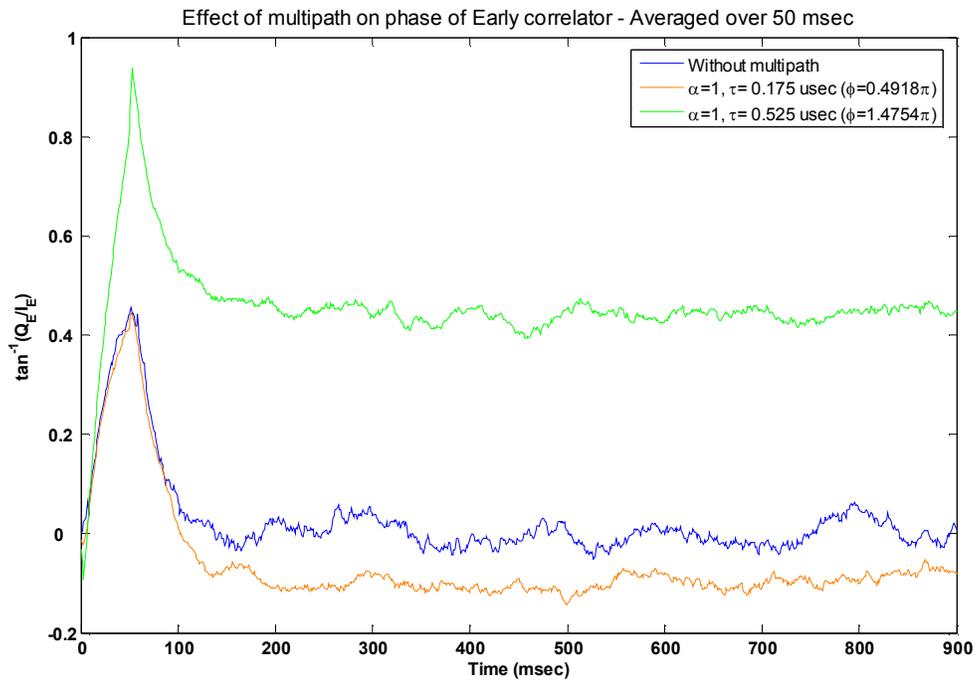


Figure 13. Multipath effect on phase of early correlator output for PRN 7 in presence of 18 dB noise and 7 other equally strong satellite signals (PRN: 3, 12, 15, 19, 23, 28 & 31) – Averaged over 50 msec

Figure 14 shows the ELP plots for $\Phi \approx \pi/2$ and $\Phi \approx 3\pi/2$ in presence of 18 dB stronger noise

and 7 other satellite signals. It can be seen that the margin for multipath detection is much better in this case as compared to one in Figure 13. In order to further confirm the effectiveness of this proposed variable, histograms of averaged ELP are shown in Figure 15 and Figure 16 for $\Phi \approx \pi/2$ and $\Phi \approx 3\pi/2$. It can be seen that presence of multipath signal is easily distinguishable in both the cases, although the margin for $\Phi \approx 3\pi/2$ is even higher. As mentioned in section 3, it is because of the reason that in this case separation between I-channel peak due to LOS and Q-channel peak due to reflected signal is $0.525 \mu\text{sec}$, however it is just $0.175 \mu\text{sec}$ in case of $\Phi \approx \pi/2$. Higher time domain separation leads to wider polygons, which in turn produces more deviation of early and late phases in software receiver tracking results.

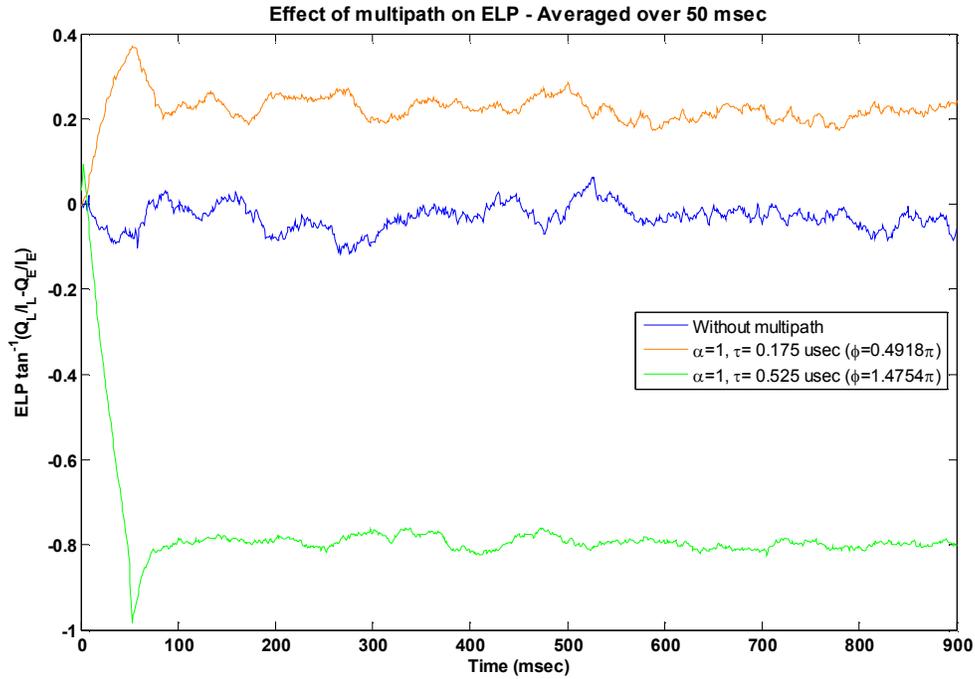


Figure 14. Multipath effect on ELP output for PRN 7 in presence of 18 dB noise and 7 other equally strong satellite signals (PRN: 3, 12, 15, 19, 23, 28 & 31) – Averaged over 50 msec

4.3 ELP limitations

Although the plots above shows that a threshold on ELP variable could provide good multipath detection scheme, however these are initial results and still has few limitations before it can be put to test in real environments. Some of them are:

- i. Only single multipath signal has been considered in this paper. The analysis would be more complex with multiple reflections.
- ii. It is not good in detecting multipath when phase difference between LOS and reflected signal is π or its multiples. However, this problem may be sorted out with introduction of new civilian signal L2C. Assuming L1 and L2C are both reflected through same surface and followed same path before reaching receiver, they can aid each other as they have different carrier frequencies. If ELP is monitored for both L1 and L2C, there are very less chances that in case of multipath occurrence, the reflected signal and LOS have phase difference of π or its multiples for both L1 and L2C.

- iii. The algorithm to detect multipath on the basis of ELP values assumes that the ELP threshold in absence of multipath is already known. However, this value may change in varying SNR and cross-correlation conditions. Extensive experiments and thorough analysis is required to update this threshold depending on current conditions.

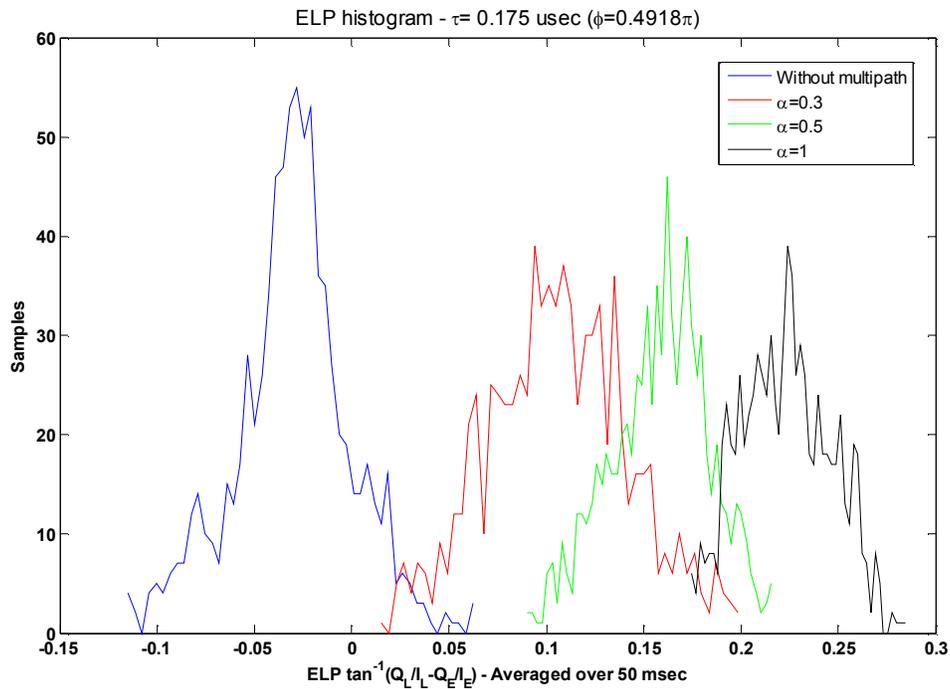


Figure 15. Histogram for ELP output averaged over 50 msec of PRN 7 in presence of 18 dB noise and 7 other equally strong satellite signals (PRN: 3, 12, 15, 19, 23, 28 & 31) – $\Phi \approx 0.4918\pi$

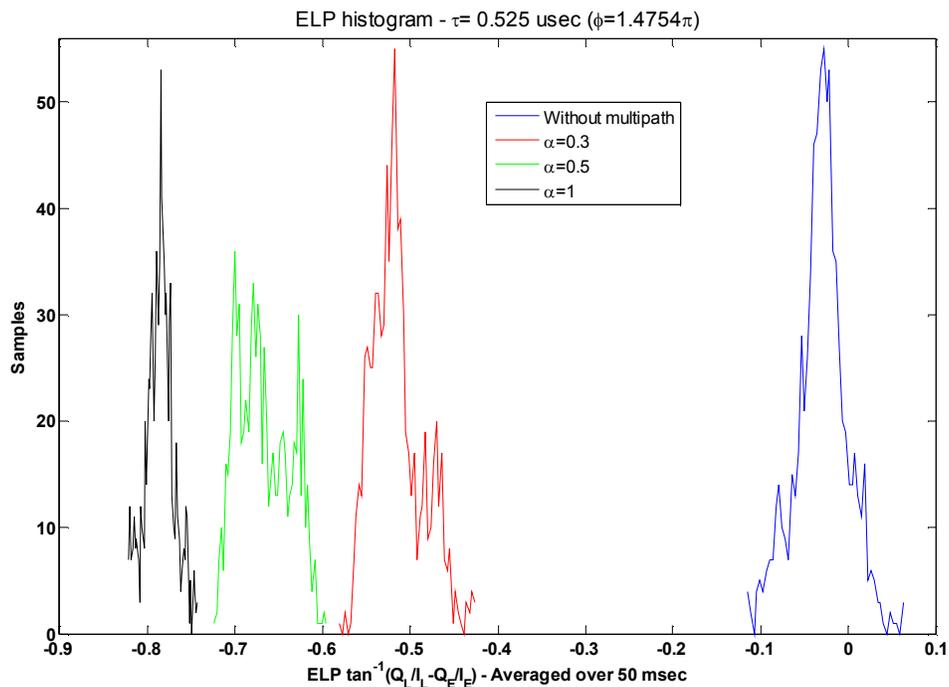


Figure 16. Histogram for ELP output averaged over 50 msec of PRN 7 in presence of 18 dB noise and 7 other equally strong satellite signals (PRN: 3, 12, 15, 19, 23, 28 & 31) – $\Phi \approx 1.4754\pi$

5. CONCLUSIONS

This paper has presented analysis of phase variations due to the presence of multipath. The analysis of I and Q-channel outputs of a correlator in a GPS receiver led to finding of a novel variable, named early late phase (ELP). It has been shown that this variable can be used in estimating multipath, however there are few limitations to be addressed in order to make it robust for all real time scenarios. Further research would be carried out in future to achieve that goal.

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