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COMPOSITE DATA WEIGHT ANALYSIS OF IONOSPHERE MODEL DETERMINATION

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ABSTRACT

This paper presents an idea to establish the region ionosphere model using Bayes least square with the circumstance that lacking of real observation data in special region. Besides real observation data, data calculated by Global Ionosphere Model (GIM) has been used as a background. Weights of GIM data and real observation data should be estimated with a standard weight before using them because accuracies of these data were different. So weights intra two kinds of data have been discussed with the covariance matrix respectively at first. And then weight inter two kinds of data have been analysed, an integer weight factor has been put forward for the determination of the weight inter two kinds of data. Lastly, the feasibility of the idea and the method to estimate the data weight were proved by an investigation scenario. The accuracy of the model established using this idea reaches 85% in the region where has none real observation data, which make it extraordinary suitable for precisely region satellite navigation data simulation purposes.

KEYWORDS: Global Ionosphere Model, Real Observation Data, Data Weight, Polynomial Ionosphere Model.

1. INTRODUCTION

The ionosphere delay is one of the largest sources of error that affects the positioning accuracy of any satellite positioning system. Much effort has been made in establishing models to make this error as small as possible. These models vary in accuracy, input data and computational complexity. The choice between the different models depends on the individual circumstances of the user. From the precisely region satellite navigation data simulation point of view, the aim was to select a model for the simulation of satellite navigation data that gives a good description of the ionosphere's nature with a high degree of accuracy in computing the ionospheric delay that would yield more precisely simulated data. Real-time precisely region ionosphere model that established with real observation data was usually chosen to calculate the Total Electron Content (TEC) in this circumstance, such as polynomial ionosphere model (Jingnan Liu, 1999), if the real observation data was insufficient to establish the real-time ionosphere model, GIM would be used alternatively.

GIM is produced by Centre for Orbit Determination in Europe (CODE) to describe the global TEC maps (Ashraf, F., 2002; Schaer, S., 1998a). It is presented by a spherical harmonic expansion of degree 15 and order 15 referring to a solar-geomagnetic frame. It gives a good description and visualization of the behaviour of the global ionosphere with large space and time resolution. Because the degree of the spherical harmonic expansion used in GIM is only 15, the space resolution considered is $360^\circ/15 = 24^\circ$ (Stefan, S., 1998b), which is not small enough to describe more details of the ionosphere performance. Further more, affected by the distribution of the IGS (International GPS Service) ground stations, GIM does not give correction for the ionospheric delay to a good degree of accuracy, which only reaches about 70% in the region of China (Hongping Zhang, 2006). So the real-time region ionosphere model would be indispensable to be used for higher quality standard correction with respect to the GIM in this region. Polynomial ionosphere model is one of the real-time region ionosphere models. Accuracy of this kind of model is affected mainly by density and distribution of real observation data. In the region with sufficient real observation data, Polynomial ionosphere model presents a perfect performance to correct the ionospheric delay, but the shortcoming of this model is that it is too strong depending on the real observation data, it can not maintains its good degree of accuracy in the region with insufficient real observation data, moreover, it would even give some negative TEC in the blankness region where has none real observation data (Hongping Zhang, 2006).

The authors based on these analyses have developed a method to establish polynomial ionosphere model, which GIM data that calculated by GIM and real observation data as the whole original measurement. Combining these two kinds of data as a whole has tow advantages with respect to use them respectively: Firstly, GIM data could give a complementarity to real observation data and clear up the blankness region where has none real observation data; Secondly, real observation data could remedy GIM's low space resolution and low accuracy in this region. And then, weights of these two kinds of data in data processing with Bayes least square have been discussed, which involved both intra and inter weights of them.

2. MEASUREMENT EQUATION OF COMBINED MODEL

Polynomial ionosphere model uses the following equation (1) to compute Vertically TEC (VTEC) in different latitudes and with different times.

$$VTEC = \sum_{i=0}^{n1} \sum_{k=0}^{m1} E_{ik} (\varphi - \varphi_0)^i (s - s_0)^k \quad (1)$$

$$s - s_0 = (\lambda - \lambda_0) + (t - t_0)$$

where: $VTEC$ = Vertically Total Electron Content
 $n1, m1$ = degree of model
 E_{ik} = coefficients of model
 φ_0, λ_0 = geomagnetic latitude and longitude of model expansion point
 φ, λ = geomagnetic latitude and longitude of pierce point
 t_0 = central time of the model effective interval
 t = local time

GIM uses the following equation (2) to compute VTEC in different latitudes and with different times.

$$VTEC = \sum_{n=0}^{n_{max}} \sum_{m=0}^n \tilde{L}_{nm}(\sin\varphi)(\tilde{C}_{nm} \cos(ms_s) + \tilde{S}_{nm} \sin(ms_s)) \quad (2)$$

$$s_s = \lambda - \lambda_s$$

where: n_{max} = the highest degree of spherical harmonic expansion
 \tilde{L}_{nm} = standardized Legendre function of degree n order m
 $\tilde{C}_{nm}, \tilde{S}_{nm}$ = coefficients of spherical harmonic expansion
 λ_s = geomagnetic longitude of the Sun

Difference from the real observation data, GIM data does not contain the instrumental biases of satellite and receiver, then replacing $VTEC$ in equation (1) with equation (2), the measurement equation of GIM data could be gotten as following equation (3):

$$\sum_{i=0}^{n1} \sum_{k=0}^{m1} E_{ik} (\varphi - \varphi_0)^i (s - s_0)^k = \sum_{n=0}^{n_{max}} \sum_{m=0}^n \tilde{L}_{nm}(\sin\varphi)(\tilde{C}_{nm} \cos(ms_s) + \tilde{S}_{nm} \sin(ms_s)) \quad (3)$$

With respect to GIM data, VTEC are derived from double difference pseudo observation as following equation (4):

$$VTEC = 9.52437 \cdot \cos(z') \cdot (P_{i_2}^j - P_{i_1}^j + \Delta q^j + \Delta q_i) \quad (4)$$

where: z' = zenith distance
 $P_{i_2}^j, P_{i_1}^j$ = double frequency pseudo observations
 Δq^j = instrumental biases of satellite
 Δq_i = instrumental biases of receiver
 j = the number of satellite
 i = the number of receiver

The conversion of line-of-sight TEC to VTEC is done depending on a single-layer model put forward by Schaer. S (1999). Replacing $VTEC$ in equation (1) with equation (4), the measurement equation of real observation data could be gotten as following equation (5):

$$\sum_{i=0}^{n1} \sum_{k=0}^{m1} E_{ik} (\varphi - \varphi_0)^i (s - s_0)^k - 9.52437 \cdot \cos(z') \cdot (\Delta q^j + \Delta q_i) = 9.52437 \cdot \cos(z') \cdot \Delta P_{i_21}^j \quad (5)$$

$$\Delta P_{i_21}^j = P_{i_2}^j - P_{i_1}^j$$

Combining equation (3) and equation (5), the whole measurement equation could be gotten as following equation (6) which uses not only real observation data but also GIM data.

$$\begin{cases} \sum_{i=0}^{n_1} \sum_{k=0}^{m_1} E_{ik} (\varphi - \varphi_0)^i (s - s_0)^k = \sum_{n=0}^{n_{\max}} \sum_{m=0}^n \tilde{L}_{nm} (\sin\varphi) (\tilde{C}_{nm} \cos(ms_s) + \tilde{S}_{nm} \sin(ms_s)) \\ \sum_{i=0}^{n_1} \sum_{k=0}^{m_1} E_{ik} (\varphi - \varphi_0)^i (s - s_0)^k - 9.52437 \cdot \cos(z') \cdot (\Delta q^j + \Delta q_i) = 9.52437 \cdot \cos(z') \cdot \Delta P_{i21}^j \end{cases} \quad (6)$$

3. ANALYSIS OF DATA WEIGHT

Data weight should be analysed because of the different accuracy of GIM data and real observation data while combining them together as a whole to establish polynomial ionosphere model using the method of Bayes least square. Analyses involved both intra and inter weights of GIM data and real observation data. Firstly, intra weight has been studied with the covariance matrix respectively, then an integer weight factor, which based on the accuracy of GIM data with respect to real observation data, has been put forward for inter weight of these two kinds of data. More detailed descriptions of the weight analysis are presented as following.

3.1 Weight analysis intra GIM data

Known as equation (2), GIM data only depend on the geomagnetic latitude and longitude of pierce point for the same model degree and coefficients, different GIM data would be computed in different pierce point. So, according to covariance propagation theory, GIM data are independent and their covariance equal to zero because of the independence of different pierce points. File of GIM coefficients provided by IGS analysis centers not only includes GIM coefficients but also the corresponding variance of GIM coefficients. Once having variance of GIM coefficients, the variance of GIM data could be computed according to covariance propagation theory as following equation (7):

$$\sigma_i^g = \sum_{n=0}^{n_{\max}} \sum_{m=0}^n \tilde{L}_{nm} (\sin\varphi_i) (\sigma(\tilde{C}_{nm}) \cdot \cos(ms_{si}) + \sigma(\tilde{S}_{nm}) \cdot \sin(ms_{si})), \quad i = 1 \cdots n_g \quad (7)$$

where: σ_i^g = variance of GIM data
 i = the serial number
 n_g = the total number of GIM data
 $\sigma(\tilde{C}_{nm})$ = variance of coefficient \tilde{C}_{nm}
 $\sigma(\tilde{S}_{nm})$ = variance of coefficient \tilde{S}_{nm}

The minimal variance was chosen as the standard weight variance after computing out all variance of GIM data, then weight intra GIM data could be calculated with respect to this standard weight variance, and the weight matrix intra GIM data could be described as following equation (8):

$$W^g = \begin{bmatrix} w_1^g & 0 & \cdots & 0 \\ 0 & w_2^g & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & w_{n_g}^g \end{bmatrix} \quad w_i^g = \left(\frac{\sigma_0^g}{\sigma_i^g} \right)^2 \quad (8)$$

where: W^g = weight matrix intra GIM data
 w_i^g = weight of GIM data
 σ_0^g = standard weight variance

3.2 Weight analysis intra real observation data

The code noise of receiver is the main factor that affects the accuracy of VTEC which derive from double difference pseudo observation. For the purpose to improve the accuracy of this kind of VTEC, time-differenced code minus-carrier has been used to remove the effect of code noise (Jingnan Liu, 1999). After the smooth process with time-differenced code minus-carrier, real observation data were independent with a same accuracy degree, so their covariance equal to zero and variance is same. Based on this analysis, the weight matrix intra real observation data could be described as following equation (9):

$$W^r = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \\ 0 & \dots & & 1 \end{bmatrix}_{n_r} \quad (9)$$

where: n_g = the total number of GIM data
 W^g = weight matrix intra GIM data

3.3 Weight analysis inter both data

After got intra weight matrixes of both data, the following problem is how to unite them so that two kinds of data could be combined as a whole. Integer weight factor has been put forward as the weight inter two kinds of data, integer weight factor is a index that presents a integer description for the GIM performance in the estimation of the ionosphere delay which derived from real observation data. Two methods have been study for the determination of integer weight factor.

One of the methods is take an experiential accuracy index as the integer weight factor, this experiential index is estimated based on long-term and abundant compare and analysis between GIM data and real observation data in special region, which represents the statistical integer accuracy of GIM data. With this experiential integer weight factor, weight matrixes intra these two kinds of data could be united to a whole weight matrix W_1 ; Another method to determine integer weight factor is a real-time method with respect to the first one, GIM data and real observation data were divided into small parts with 2 hours time resolution. Then the real-time accuracy of GIM data in each 2 hours time interval could be determine as the real-time integer weight factor after the compare and analysis between GIM data and real observation data in corresponding time interval. Using these real-time integer weight factors, weight matrixes intra these two kinds of data could be united to a whole weight matrix W_2 , more details about W_1 and W_2 is described as following equation (10):

$$W_1 = \begin{bmatrix} \alpha^2 \cdot W^g & \\ & W^r \end{bmatrix}_{(n_g+n_r)} \quad ; \quad W_2 = \begin{bmatrix} \alpha_1^2 \cdot W_1^g & & & \\ & \alpha_2^2 \cdot W_2^g & & \\ & & \ddots & \\ & & & \alpha_k^2 \cdot W_k^g \\ & & & & W^r \end{bmatrix}_{(n_g+n_r)} \quad (10)$$

where: α = experiential integer weight factor
 α_i = real-time integer weight factor of each time interval
 W_i^s = intra weight of GIM data of each time interval
 i = serial number of time interval
 k = the total number of time interval

4 INVESTIGATION SCENARIO & DISCUSSION

The scenario tests the feasibility of the method to establish polynomial ionosphere model, and then the model accuracy in the blankness region where has none real observation data has been analysed.

The scenario involved processing three original data: IGS precise ephemeris provided by The IGS analysis centers in SP3 format, coefficients of GIM produced by CODE as the final product of global ionosphere maps and real GPS observation data provide by GPS Crustal Movement Monitoring Net in China in RINEX format. The testing day was 11th of May 2004 and the testing region is within longitude form 70 to 145 and latitude from 10 to 55. Figure 1 shows the distribution of pierce points of the testing day in the testing region. All the pierce point were divided into two parts in the test, the first part whose longitude of the pierce point is less than 130 degree was used as the original measurements to establish model, the second part whose longitude of the pierce point is more than 130 degree was used as standard data to test the accuracy of model in blankness region where longitude is from 130 to 145.

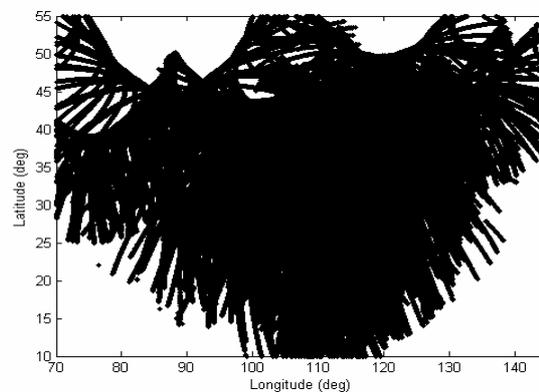


Figure 1. Distribution of pierce point

Learning from the result of test, mean bias of the model with experiential or real-time integer weight factor are 2.34 TECu and 2.36 TECu respectively, and model accuracies reach 85.82% and 85.70% respectively in blankness region. More detail about the bias of the zenithal ionospheric delay between the model and standard data are shown as figures 2 and 3, figure 2-1 and 3-1 show bias value, and the percentage of bias are shown in figure 2-2 and 3-2. It can be concluded that the method to establish polynomial ionosphere model with experiential or real-time integer weight factor is feasible, this method remedies the shortcoming of region ionosphere model's strong depending on the real observation data, and the model established by this method gives higher accuracy in computing the ionospheric delay in the blankness region with respect to both GIM and region ionosphere model that established only using real observation data.

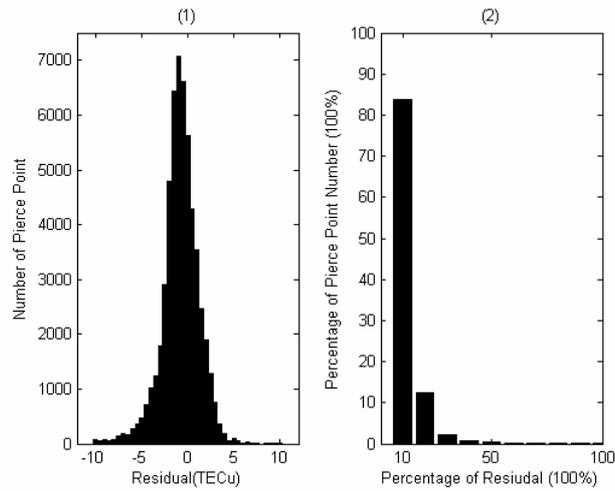


Figure 2. Bias information with exponential integer weight factor

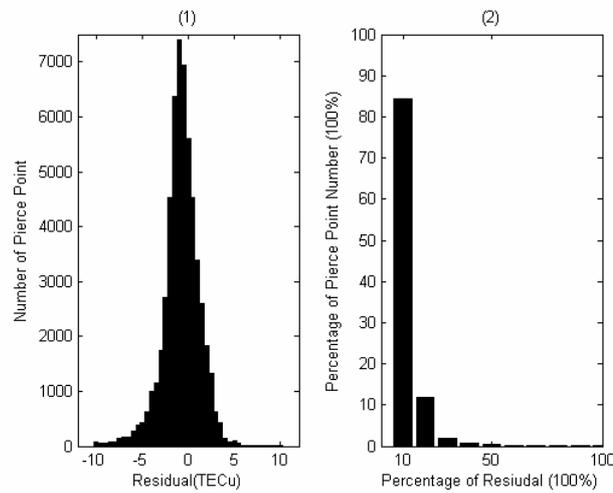


Figure 3. Bias information with real-time integer weight factor

5 CONCLUSION

Combining GIM data and real observation data as a whole could clear up the blankness region where has none observation data firstly, and remedy GIM's low space resolution and low accuracy in special region secondly. The accuracy of the polynomial ionosphere model established using the combining data reaches 85% of the total effect even in the blankness region, which make it extraordinary suitable for precisely region satellite navigation data simulation purposes.

The effect of exponential integer weight factor and real-time integer weight factor are approximate, both of them could unite GIM data and real observation data as a whole effectively.

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