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Unequally Displaced Correlator in Tracking-loop for Multipath Mitigation

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ABSTRACT

Multipath interference affects the accuracy of the pseudorange as well as the carrier phase measurement in the tracking-loop of a GNSS receiver. In high precision applications, *RTK* for example, the multipath error is the dominant component of the error sources.

Many efforts have been made for the mitigation of the multipath errors. By examining the sampling ‘arms’ of the correlation peak/peaks (e.g., ‘early’ & ‘late’ arms) in the tracking-loop, it is the fact that the multipath signals are the late arrival to contaminate the correlation profile. It suggests that one should treat the ‘early’ and the ‘late’ arms differently in the delay estimation in order to reduce the impact of the multipath interference.

The idea of *unequally displaced correlator* means that in a tracking-loop, the ‘early’ and the ‘late’ arms (or other type of sampling methods) are not necessary to be equally located around the central position of *LOS* (the Line-Of-Sight signal). In order to reduce the multipath error, in principles, the ‘late’ arm should be placed more close to the centre of *LOS* to avoid being contaminated. This is different from the classical equal spacing for both ‘early’ and ‘late’ arms.

Analysis as well as simulations has been done with the GPS C/A code tracking process. The results show clearly improvement in the multipath environment, and it is more stable comparing with, for example, the so-called ‘narrow correlator’ approach.

KEYWORDS: Delay estimation, Tracking-loop, multipath.

1. INTRODUCTION

One of the biggest problems affecting the accuracy of a positioning and navigation system is the multipath interference. The measurement error comes initially from the delay estimation process of the tracking loop in the receiver. The multipath components contaminate the delay profile to give error to the delay estimation and then to reduce the accuracy of the pseudorange as well as the carrier phase measurement. In high precision applications (e.g., differential GNSS, RTK, mRTK,), the multipath error is the dominant component of the error sources.

Many efforts have been made for the mitigation of the multipath error. One effective approach was the so-called ‘narrow correlator’. The art of the technique is that comparing with classical one-chip spacing method for the delay estimation; it employs narrower spacing between the ‘early’ and ‘late’ arms on the shoulder of the correlation peak. The idea is clear that by narrowing the estimation window, the multipath impact can be reduced due to the different time-of-arrival between *LOS* (the Line-Of-Sight signal) and the multipath(s). On the other hand, narrowing the estimation window will reduce the dynamic boundary of the tracking range. The penalty is the reduction of the tracking stability. It is not always acceptable especially in the weak signal situation.

As we know that the multipath signals are the late arrival to contaminate the delay profile. The influence is higher to the ‘late’ shoulder than to the ‘early’ shoulder of the *LOS* peak. For this reason, one should treat the ‘early’ and the ‘late’ differently in the delay estimation to combat against the multipath interference.

In this report, a method of *unequally displaced correlator* is introduced. In the delay estimation process of the tracking loop, the ‘early’ and the ‘late’ arms are not necessary to be located symmetry around the centre of the *LOS* peak. In order to reduce the multipath impact, the ‘late’ correlator is placed more close to the centre of *LOS,* while the ‘early’ arm can be placed more distances away to keep a relatively wider estimation window or tracking range. Figure 1 shows the different sampling arrangement for the *classical*, the *narrow* and the proposed *unequally displaced correlator*.

Initial study and simulation work in this report is based on the GPS C/A code. The results show that this is an effective and simple method for the multipath mitigation. It is an optimum choice for the tracking process considering both the tracking stability and the multipath effect especially in the weak signal environment.

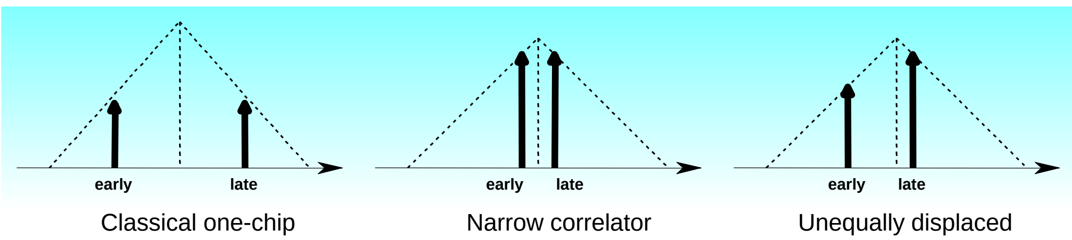

Figure 1. The ‘early’ and ‘late’ location in different approaches

2. SIGNAL AND TRACKING LOOP MODEL

2.1. Signal model

Let's start with the channel description for the received signal model. Suppose we have a signal in which the number of multipath components is M, the received spread-spectrum signal in the base band can be written as:

$$x(t) = \sum_{i=0}^{M} a_i c(t - \tau_i)\cos(\omega_i t + \theta_i) + n(t) \tag{1}$$

where c is the spread-spectrum code and a_i, τ_i, ω_i and θ_i are the time dependent amplitude, delay, frequency and phase of the i-th signal, respectively. For simplicity, the time dependence of the parameters is left out. $n(t)$ is the noise.

2.2. Receiver model

Figure 2 shows the principle diagram for the delay locked loop (DLL) in the tracking process. The received signal is first down converted to the baseband (***I*** and ***Q***) and then multiplied (correlated) by the 'early' ($\mathbf{c}_E$) and 'late' ($\mathbf{c}_L$) codes separately for 'early' and 'late' arms. The results are then integrated over a certain time. The result goes through the loop filter then to NCO for the local code generator control.

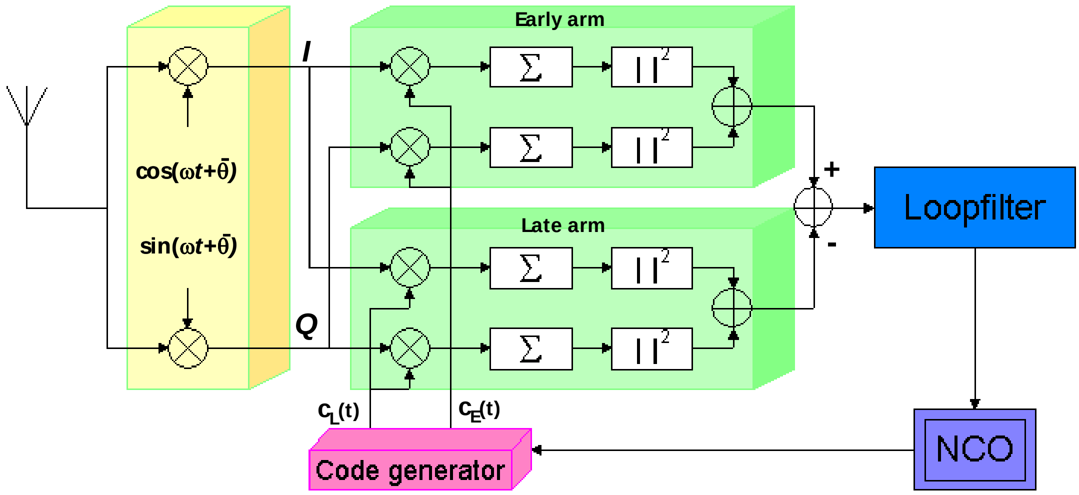

Figure 2. Tracking loop for the delay estimation (DLL)

In Figure 2, the non-coherent DLL is presented. Actually all the analysis in this report is true for the coherent DLL as well. The correlation function R(τ) of the spread-spectrum code $c(t)$ is simplified to be a triangle for the C/A code:

$$R(\tau) = \begin{cases} 1 - |\tau / T_c| & \textbf{for } |\tau| < T_c \\ 0 & \textbf{elsewhere} \end{cases} \tag{2}$$

where T_c is the chip interval of the spread-spectrum code.

2.3. S-function

In order to analysis the tracking loop performance, the S-function (S-curve) is introduced. It is the result of the subtraction of the 'early' and 'late' versus the estimated delay error $\hat{\tau}$. The S-function for equally displaced correlator (the classical one-chip spacing as well as the narrow correlator) for 'early' and 'late' is defined as:

$$S(\hat{\tau}_0) = C\sum_{i=0}^{M} a_i \cos\left(\theta_i - \hat{\theta}_0\right)\left\{R\left(\hat{\tau}_0 - \tau_i + T_c d/2\right) - R\left(\hat{\tau}_0 - \tau_i - T_c d/2\right)\right\} \tag{3}$$

where d is the early-late spacing and $\hat{\theta}_0$, $\hat{\tau}_0$ are the estimates of the line-of-sight carrier phase and delay respectively, C is the normalization constant.

For example, if there is only *LOS* (M = 0) and the signal amplitude is 1 (a_0 = 1). The S-function to track *LOS* is simplified to as:

$$S(\hat{\tau}) = C_0\left\{R\left(\hat{\tau} + T_c d/2\right) - R\left(\hat{\tau} - T_c d/2\right)\right\} \tag{4}$$

S-function can dynamically describe the tracking loop performance. The tracking process works on the linear region of the S-curve towards the zero crossing and a negative derivative to guarantee the convergent of the process:

$$\frac{\partial S(\hat{\tau})}{\partial \hat{\tau}} < 0 \tag{5}$$

In the region of positive derivative, the system is unstable. The normalization constant C in Equation 3 is to let the derivative of the S-curve be –1 in the linear region in order to have a linear feed back in the tracking loop. That is:

$$\frac{\partial S(\hat{\tau})}{\partial \hat{\tau}} = -1 \tag{6}$$

2.4. Multipath error analysis

The study of the multipath effect is normally focused on the situation of just one multipath signal, so M = 1 in Equation 3. Because the phase difference is also the matter of the influence, two extreme cases are studied: the in-phase and the out-of-phase. The in-phase means that the phase of the multipath signal is the same as that of *LOS* (0 degree), while the out-of-phase is just the opposite (180 degree).

The study is normally started with the S-function. Due to the multipath signal impact, the zero crossing spot gives the combined effect both by *LOS* and the multipath(s). It turns out to be the measurement error of the delay estimation.

3. UNEQUALLY DISPLACED CORRELATOR

Based on the fact that the multipath signals present on the 'late' shoulder of the correlation peak of *LOS*, the multipath influence to *LOS* is bigger on the 'late' shoulder than on the ‘early’ shoulder. In other word, the sampling value from the ‘early’ shoulder is more reliable for the delay estimation. When arranging the position for sampling the delay profile, one should consider locating the ‘late’ arm(s) to be more closed to the centre of the correlation peak of *LOS* to reduce the impact by the multipath signal.

3.1. Modified S-function

A general S-function is derived for the tracking process. In Equation 7, the amplitude of the signal is set to be 1 and the normalization factor is set to let the derivative on the linear region of the S-function equal to –1 (Equation 6). The correlation function R(τ) is supposed to be a triangle as in Equation 2. Of course, the S-function can be modified for other kind of shape of the correlation function as well.

$$S(\hat{\tau}) = \frac{R(T_c d_E) R(T_c d_L)}{R(T_c d_E) + R(T_c d_L)} \left\{ \frac{R(\hat{\tau} + T_c d_E)}{R(T_c d_E)} - \frac{R(\hat{\tau} - T_c d_L)}{R(T_c d_L)} \right\} \qquad (7)$$

where d_E, d_L are the displacement from the central of the correlation peak to the ‘early’ arm and the ‘late’ arm, respectively.

For the equally displaced correlator, d_E, = d_L. Equation 7 turns out to be the same as in Equation 4. For the unequally displaced correlator, d_E and d_L are not necessarily to be the same. In order to reduce the multipath influence appearing on the ‘late’ arm, it is suggested that one should let $d_L \leq d_E$.

Equation 7 is for one path signal only. For more signals (*LOS* + multipath), the equation can be simply overlaid (sum up) for multipath analysis.

Let’s make the analysis for the tracking loop performance using the three different approaches of the correlator displacement discussed in this report (see Figure 3).

The three different approaches are: the classical one-chip correlator displacement (case A, blue line in Figure 3), the narrow correlator (case B, green line) and the unequally displaced correlator (case C, red line). In the classical case, the displacement of the early–minus-late is one chip and equally spaced (marked as 0.5–0.5 in Figure 3) and symmetry to the centre. In the narrow correlator case, the early–minus-late is one fifth of a chip and also equally spaced (marked as 0.1–0.1). In the unequally displaced correlator case, the ‘early’ arm is placed at 0.5 chips and the ‘late’ arm at 0.1 chips to the correlation peak (marked as 0.5–0.1).

We see that for the classical one-chip case, the linear region on the S-curve is from A1 to A2. The negative derivative region is also from A1 to A2. It means that the tracking process is convergent in this region. In the narrow correlator case, the linear and the convergent region are from B1 to B2. In the unequally displaced correlator case, the linear region is from C1 to C2 while the convergent region is from C1 to C3. The tracking range from C1 to C3 is much bigger than that from B1 to B2 in the narrow correlator case. From this analysis, it is clear that the tracking performance of unequally displaced correlator is better than the narrow correlator. It will be more robust against the code delay error and the noise comparing with the narrow correlator approach.

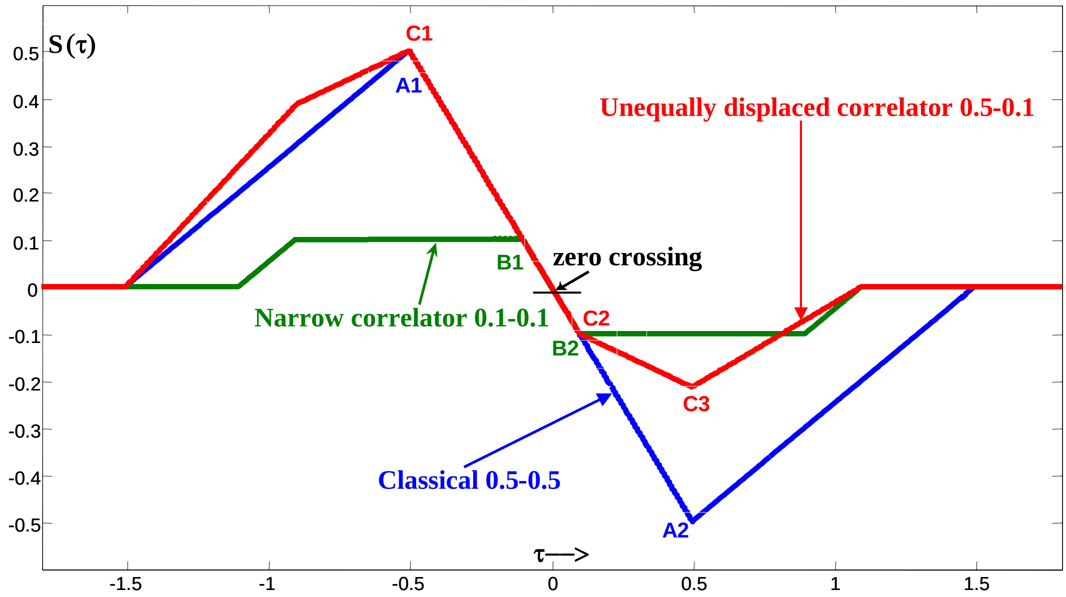

Figure 3. S-functions for different correlator displacement

3.2. Multipath errors

Multipath error analysis is performed using the GPS C/A code for different cases. As mentioned in Chapter 2.4, the analysis is based on one multipath signal (M = 1) and the in-phase and out-of-phase situations are considered as two extreme cases for the error envelope. SMR (line-of-sight signal to multipath signal ratio) is set to be 6 dB (the amplitude of the multipath signal is half of that of *LOS*).

For the comparison reason, three cases are chosen for the analysis: the classical one-chip correlator (0.5-0.5), the narrow correlator (0.2-0.2) and the unequally displaced correlator (0.8-0.2). The reason using '0.8-0.2' as the unequally displaced case is that the total displacement between the 'early' arm and the 'late' arm is 1 chip that is comparable with the classical one-chip case, while 0.2 for the 'late' arm is comparable with the selected narrow correlator case.

Figure 4 shows the analytical results of the multipath errors for the three different cases.

From the results we see that unequally displaced correlator approach can reduce the multipath error as the narrow correlator does. At the far-delay region, when the delay of multipath signal is more away from the central peak, the performance by the unequally displaced correlator is clearly better than that by the narrow correlator.

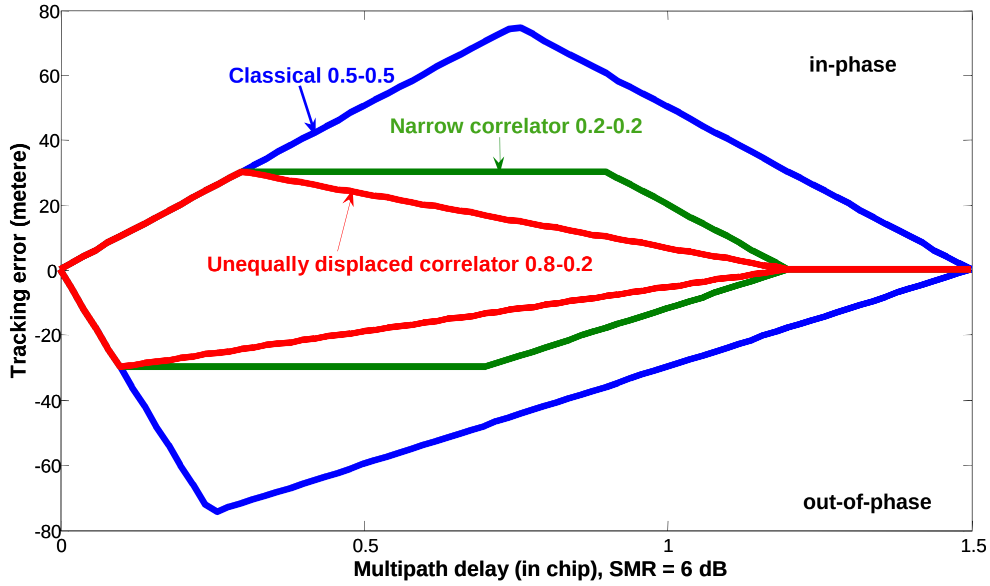

Figure 4. The multipath error analysis

3.3. The DLL discriminator

The DLL discriminator for the delay estimation in the tracking loop can be easily derived from the S-function. For example, based on equation (7), the non-coherent DLL discriminator for both the equally and the unequally displaced early-minus-late correlator is:

$$DLL = \frac{1}{A+B}\left(A \times R(T_c d_L) - B \times R(T_c d_E)\right) \quad \text{(in chips)} \tag{8}$$

where A and B are the estimated amplitudes of the early-arm and the late-arm respectively.

For the equally displaced correlator, $R(T_c d_L)$ and $R(T_c d_E)$ are the same. Let's use a constant C to replace them; we get the familiar DLL discriminator:

$$DLL = C\left(\frac{A-B}{A+B}\right) \quad \text{(in chips)} \tag{9}$$

4. SIMULATIONS

Computer simulations have been carried out with the GPS C/A code (sv = 18). The satellite signal power level is set to be 50 dB-Hz and the signal to the multipath ratio (SMR) is set to be 6 dB. The Doppler frequency is set to be 1000 Hz. Both in-phase and out-of-phase of the multipath are considered. In the simulations, a pure signal (without multipath) is first processed as the reference signal and the pseudorange estimation is done based on the sampling counter and the NCO memory in the tracking loop of the receiver. In all other cases, the pseudorange estimation results are used to compare with the reference to get the estimation error. The pseudo-range estimation error is the mean value over 4 second's period after the tracking loop is in a stable stage.

The simulation results are shown in Figure 5. Three different cases mentioned in Chapter 3.2 are all adopted for the simulation. As for the comparison, the analytical results are also shown in the figure as the dashed lines while the simulation results are shown as '*' marks.

It clear shows that the simulation results are in good agreed with the analysis. In this special situation, the results by the unequally displaced correlator method are better than those by all other cases especially at the far-delay region.

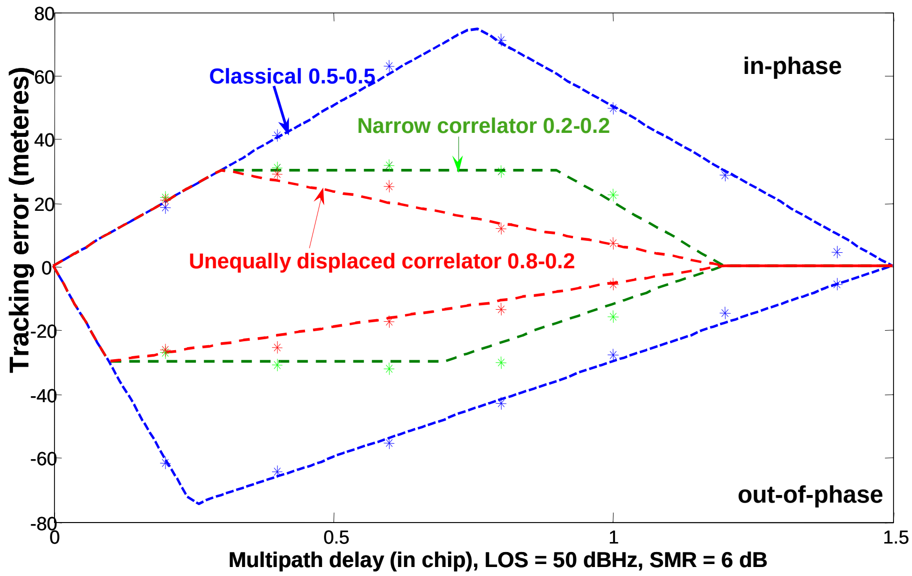

Figure 5. Aanalysis vs. the simulation results

5. CONCLUSION

In the tracking process, the delay estimation is the 'shape' related. The 'shape' is the auto-correlation function of the spread-spectrum. The classical 1-chip symmetrical arrangement for the 'early' and 'late' arms when sampling the 'shape' is based on a general consideration in a normal situation. It is not an optimum solution for different situation, for example, when the multipath(s) presents.

In a multipath environment, the sampling position in the correlation function should be moved to be close to the peak centre to reduce the multipath impact. Based on the fact that multipath signal is late arrival to contaminate *LOS*, one should consider locating the 'late' arm to be more close to the centre. An unsymmetrical arrangement is proposed in this report.

In our initial study, the S-function for the tracking-loop performance is derived. It is for an arbitrary arrangement of the 'early' and 'late' arms based on the normal triangle correlation function. For other type of correlation function and sampling methods, the theory can also be applied using a modified S-function.

In both narrow correlator approach and unequally displayed correlator approach, when one try to locate the sampling arms to close to the centre, two benefits can be achieved. One is the improvement of the *SNR* for the delay estimation; another one is the mitigation of the multipath interference. But the tracking process will get less stable due to the tracking range reduction of the negative derivative. It is the problem for tracking a weak signal or if there is the burst-like noise exists. From this point of view, the unequally displaced correlator has clearly advantage over the simple symmetrical narrow correlator because the total displacement between the early and the late could be wider. It will be more robust, without sacrifice the benefit of the multipath mitigation at the same time.

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