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A Framework for Efficient Cooperative Localization with Non-Gaussian Ranging Error Distributions

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ABSTRACT

Cooperative localization takes advantage of the range measurements between neighbouring agents to improve both the availability and accuracy of positioning systems. Distributed belief propagation is a promising technique for data fusion in cooperative localization. Difficulties with belief propagation lie in reducing the required communication overhead and computational complexity. Most of the existing works on this subject are based on Gaussian ranging error models, which are fine for outdoor applications but not suitable for harsh propagation environments such as indoor and urban areas. In this paper, a framework for efficient cooperative localization is proposed, which can be applied to systems with non-Gaussian ranging error distributions. The communication and computational cost is reduced by passing approximate beliefs represented by Gaussian distributions between neighbours and by using an analytical approximation to compute peer-to-peer messages. The proposed scheme is validated experimentally on a deployed indoor localization system, and is shown to achieve high accuracy with low communication and computational cost.

KEYWORDS: cooperative localization, belief propagation, indoor positioning, ranging error distribution

1. INTRODUCTION

The coverage and accuracy of localization systems are impaired in harsh propagation environments due to signal blockage and large ranging errors. Cooperative localization (Wymeersch *et al.,* 2009 and Patwari *et al.,* 2005) is a promising technology that can be used under such conditions to improve the performance of localization systems. By exploiting range measurements between neighbouring mobile nodes, cooperative localization not only facilitates the positioning of nodes that do not have sufficient anchors within communication range, but also dramatically improves the positioning accuracy. In addition, cooperative localization enables rapid deployment of positioning systems: by placing cooperative nodes across the interested area, only a few anchors are required to cover a large area. Potential application scenarios of such a configuration include location services for military operations and first responders such as fire fighters.

Various approaches have been proposed in the literature for cooperative localization, including those based on deterministic optimization in Biswas *et al.* (2006) and Tseng (2007), and the multidimensional-scaling (MDS)-based approach in Costa *et al.* (2006). In addition, iterative least square (ILS) is used along with network multilateration in Patwari *et al.* (2005) to locate nodes that do not have sufficient anchors within communication range. Given the distribution of ranging errors, cooperative localization can also be achieved in a statistical framework using maximum-likelihood (ML) estimation (Wymeersch *et al.*, 2009). The most sophisticated and promising approaches that have been proposed to date are those based on distributed belief propagation (BP) (Wymeersch *et al.*, 2009, Ihler *et al.*, 2005, Savic and Zazo, 2012, Lien *et al.*, 2012), which can be implemented in a distributed fashion intrinsically and achieve high accuracy.

Since BP-based cooperative localization algorithms are based on exchanging and updating probabilistic distributions, the associated communication overhead and computational complexity can be extremely high (e.g., SPAWN in Wymeersch *et al.*, 2009), which limits their applications in practice. A popular approach for addressing this issue is to approximate the probabilistic distributions with analytical functions, which allows the estimated distributions to be efficiently exchanged and updated. In particular, it is proposed in Savic and Zazo (2012) to approximate position uncertainties with a Gaussian mixture model (GMM). Messages are approximated by a GMM in Ihler *et al.* (2005) and parametric functions in Lien *et al.* (2012) for efficient message multiplication. In Sottile *et al.* (2011), position uncertainties are represented by the traces of the corresponding position covariance matrices. A ‘linear approximation’ approach is employed in Sathyan and Hedley (2013) and Meyer *et al*. (2013) to represent node position uncertainty during message computation by increasing the variance of range measurement noise. An efficient BP algorithm (H-SPAWN) has been presented in Caceres *et al*. (2011), which uses multivariate Gaussian distributions to approximate position uncertainties and computes peer-to-peer messages analytically. An extension of the sigma point filter (SPBP) is proposed in Meyer *et al*. (2014) for efficient belief propagation. However, most of the approaches described above (including H-SPAWN, ‘linear approximation’, and SPBP) are derived based on the assumption that the ranging errors are Gaussian distributed. While it is reasonable to employ Gaussian models in line-of-sight (LOS) environments, the ranging error distributions in harsh propagation environments (e.g., indoor and urban areas) are typically non-Gaussian due to non-line-of-sight (NLOS) propagations (Wymeersch *et al.*, 2009, Alsindi *et al.*, 2009, Conti *et al.*, 2012), in which case the positioning accuracy will be compromised using Gaussian ranging error models.

In this paper, a framework for efficient cooperative localization is proposed, which is based on distributed belief propagation and can be applied to system with various types of non-Gaussian ranging error distributions. The proposed approach is more efficient than SPAWN and more general than those based on Gaussian ranging error models (H-SPAWN, SPBP etc.), which makes it suitable for localization in indoor and urban applications. The major contributions of this paper are as follows:

- An efficient cooperative localization scheme is proposed, which reduces the communication and computational cost by passing approximate beliefs represented by Gaussian distributions between neighbours and by using an analytical approximation to compute peer-to-peer messages.

- Multiple ranging error models are proposed based on ranging data collected in an indoor positioning system, which provides options for characterizing various types of ranging error distributions under NLOS conditions and facilitates analytical computation of peer-to-peer messages in the proposed cooperative localization algorithm.
- The proposed approach is validated on a real indoor localization system covering an area of over 8000 m^2, and is shown to achieve high accuracy with low communication and computational cost.

The remainder of this paper is organized as follows. Section 2 gives an introduction of cooperative localization based on distributed belief propagation, highlighting the associated communication and computational complexity. The proposed scheme for efficient cooperative localization is described in Section 3. Section 4 presents the proposed empirical ranging error models and the corresponding closed-form expressions for peer-to-peer message computation. The proposed approach is verified experimentally in Section 5 on an indoor localization system. Finally, a conclusion is drawn in Section 6.

2. COOPERATIVE LOCALIZATION BASED ON DISTRIBUTED BELIEF PROPAGATION

Consider three-dimensional (3-D) cooperative localization based on distributed belief propagation. The position of each node i at time t is denoted as $x_i^{(t)} = \left[x_i^{(t)}, y_i^{(t)}, z_i^{(t)}\right]^{\mathrm{T}}$, where $x_i^{(t)}$, $y_i^{(t)}$, and $z_i^{(t)}$ denotes the x, y, and z coordinate of the node, respectively. The range measurement between node i and node j, if available, is given by

$$r_{i,j}^{(t)} = \left\|\boldsymbol{x}_i^{(t)} - \boldsymbol{x}_j^{(t)}\right\|_2 + v_{i,j}^{(t)}, \tag{1}$$

where $v_{i,j}^{(t)}$ denotes the ranging error.

For each node i at time step t, denoting the prior distribution of its position and the set of nodes within its communication range as $\psi(\boldsymbol{x}_i^{(t)})$ and $\Gamma_i^{(t)}$, respectively, its belief is then updated by

$$b^l\left(x_i^{(t)}\right) \propto \psi\left(\boldsymbol{x}_i^{(t)}\right) \prod_{j \in \Gamma_i^{(t)}} m_{i,j}^l\left(\boldsymbol{x}_i^{(t)}\right), \tag{2}$$

in the l-th iteration of belief propagation. Note that here the belief $b^l\left(x_i^{(t)}\right)$ refers to the estimated posterior distribution of node position, while $m_{i,j}^l\left(\boldsymbol{x}_i^{(t)}\right)$ denotes the message from node j to node i, i.e., the probability distribution of node i's position given the belief of node j and the corresponding range measurement. Using $p\left(r_{i,j}^{(t)}\middle|\boldsymbol{x}_i^{(t)}, \boldsymbol{x}_j^{(t)}\right)$ to represent the likelihood associated with the range measurement between node i and j, $m_{i,j}^l\left(\boldsymbol{x}_i^{(t)}\right)$ can be computed by (see Algorithm 3 of Wymeersch *et al.*, 2009)

$$m_{i,j}^l\left(\boldsymbol{x}_i^{(t)}\right) \propto \int p\left(r_{i,j}^{(t)}\middle|\boldsymbol{x}_i^{(t)}, \boldsymbol{x}_j^{(t)}\right) b^{l-1}\left(x_j^{(t)}\right) d\boldsymbol{x}_j^{(t)}. \tag{3}$$

Each mobile node updates its belief $b^l\left(x_i^{(t)}\right)$ based on the beliefs received from its neighbours (i.e., $b^{l-1}\left(x_j^{(t)}\right)$) according to (2)(3), and broadcasts the updated belief to its neighbours for their use afterwards. This cooperative process is repeated until convergence is attained, upon which time the position of each node can be estimated from the belief. It is also worth noting that an alternative to (3) can be found in equation (9) of Ihler *et al.* (2005), which follows the standard formulation of sum product algorithm. This paper follows the formulation in Wymeersch *et al.* (2009) since it is easier to implement and has been shown therein to perform well.

The complexity of distributed belief propagation depends critically on how beliefs are represented and how messages are computed, and there are two difficulties associated with such algorithms: Firstly, the communication of beliefs (which are probability distributions) generates heavy communication overhead. Since BP algorithms take multiple iterations to converge, a large number of data packets need to be transmitted during this process, which increases the traffic load in the network even further. Secondly, the computation of messages in (3) requires computing high-dimensional integrations of non-analytic functions, which is computationally demanding and severely limits the application of BP algorithms in mobile devices. Various approaches have been proposed in the literature to address these issues. However, as has been summarized in Section 1, most of the existing approaches are derived based on the assumption that the ranging errors are Gaussian distributed, hence are suboptimal in harsh propagation environments where the ranging error distributions are non-Gaussian due to NLOS propagation. In the following section, a new framework for belief-propagation-based cooperative localization is proposed, which can be efficiently applied to systems with non-Gaussian ranging error distributions.

3. EFFICIENT COOPERATIVE LOCALIZATION WITH NON-GAUSSIAN RANGING ERRORS

Like all methods based on Monte Carlo approximation (e.g., particle filter in Arulampalam *et al.*, 2002 and SPAWN in Wymeersch *et al.*, 2009), each node maintains a set of N weighted samples (particles) in the proposed scheme, which serves as an approximation of its belief $b^l\left(x_i^{(t)}\right)$. However, to reduce the communication overhead, beliefs passed between neighbours are represented approximately using Gaussian distributions as is done[1] in Sottile *et al.* (2011), Caceres *et al*. (2011), and Xiong *et al.* (2011). The advantage of Gaussian belief approximation is twofold. Firstly, the communication overhead is reduced significantly since only few values need to be transmitted by each node per iteration. The second and more important advantage is that Gaussian belief approximation facilitates efficient computation of peer-to-peer messages, which dramatically reduces the computational complexity as will be shown in the following.

3.1 Messages Computation

Starting with the messages from anchors to tags, since the locations of anchors are exactly known, the message from anchor a to node i is simply given by

[1] It is also observed in our experiments that beliefs can be well approximated by Gaussian distributions in most situations.

$$m_{i,a}(\boldsymbol{x}_i) \propto p\big(r_{i,a} - \|\boldsymbol{x}_i - \boldsymbol{x}_a\|_2\big), \quad (4)$$

where $\boldsymbol{x}_a$ is the position of anchor a, $r_{i,a}$ denotes the ranging measurement between node i and anchor a, $p(\cdot)$ denotes the ranging error distribution (which is non-Gaussian in indoor and urban applications). Note that the indices of time and BP iterations are omitted here for simplicity of notations.

The messages from neighbouring tags (peer-to-peer messages) are computed following (3). Specifically, after receiving the belief of node j represented approximated by a Gaussian distribution with parameters $(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$, node i computes the message from node j according to

$$m_{i,j}(\boldsymbol{x}_i) \propto \int p\left(r_{i,j} - \left\|\boldsymbol{x}_i - \boldsymbol{x}_j\right\|_2\right) e^{-\frac{1}{2}(x_j-\mu_j)^T \boldsymbol{\Sigma}_j^{-1}(x_j-\mu_j)} d\boldsymbol{x}_j, \quad (5)$$

which requires computing a high-dimensional integration, for which there is generally no closed-form result. Algorithms like SPAWN (Wymeersch *et al.*, 2009) can be used in simulations to obtain the result using Monte-Carlo method, but are too computationally demanding for practical use in mobile nodes. H-SPAWN (Caceres *et al*., 2011) is very efficient but is based on a Gaussian ranging error model. An approximate peer-to-peer message computation scheme is proposed in the following, which significantly reduces the computational complexity and is applicable to non-Gaussian ranging error distributions.

Since (5) can be considered as averaging $p\left(r_{i,j} - \left\|\boldsymbol{x}_i - \boldsymbol{x}_j\right\|_2\right)$ over the uncertainty of $\boldsymbol{x}_j$, one can approximate this uncertainty by the marginal distribution of $\boldsymbol{x}_j$ along the direction to/from $\boldsymbol{x}_i$ for efficient computation of $m_{i,j}(\boldsymbol{x}_i)$. This way, the integration in (5) can be approximated by a one-dimensional (1-D) integration, which is much easier to compute. This approximation can be justified by the fact that cooperation between a pair of nodes increases the Fisher information of positions along the line connecting the two nodes, and the increment is related to the uncertainty of the nodes' positions along the line (see both Figure 4 and Remark 8 in Shen *et al.*, 2010). Figure 1 illustrates an example for two-dimensional (2-D) positioning, where the ellipse represents the uncertainty of $\boldsymbol{x}_j$ in the x-y plane and the red curve denotes the marginal distribution of $\boldsymbol{x}_j$ along the line connecting $\boldsymbol{x}_i$ and $\boldsymbol{\mu}_j$.

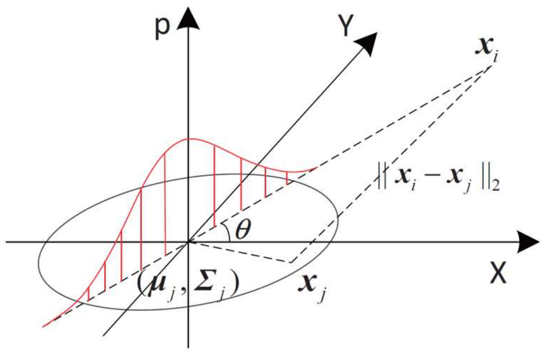

Figure 1. Approximate computation of peer-to-peer message.

Since the marginal distribution of $\boldsymbol{x}_j$ along any direction is Gaussian, $m_{i,j}(\boldsymbol{x}_i)$ can be computed approximately by convoluting the resulting Gaussian distribution with the ranging error distribution $p(\cdot)$. Specifically, the problem of message computation can be reformulated as follows: given two random variables $X_j \sim N(0, \sigma_j^2)$ and $V_{i,j} \sim p(\cdot)$, where σ_j^2 denotes the variance associated with the above-described marginal distribution of $\boldsymbol{x}_j$ (which can be computed using the method described in APPENDIX A), find the probability that $r_{i,j} = \left\|\boldsymbol{x}_i - \boldsymbol{\mu}_j\right\|_2 - X_j + V_{i,j}$. This can be obtained by

$$m_{i,j}(\boldsymbol{x}_i) \propto \int \frac{1}{\sqrt{2\pi\sigma_j}} e^{-\frac{1}{2\sigma_j^2}\left(v - r_{i,j} + \|x_i - \boldsymbol{\mu}_j\|_2\right)} p(v)dv, \tag{6}$$

the complexity of which is significantly lower than that of (5). More importantly, (6) can be evaluated using closed-form expressions for certain analytical ranging error models $p(\cdot)$ (as will be shown in Section 4), which reduces the computational complexity of (6) even further.

3.2 Proposed Cooperative Positioning Algorithm

An efficient cooperative positioning algorithm is proposed in this section based on the scheme described above. To account for the motion of moving nodes, the state of each tag i is defined to be $\boldsymbol{s}_i^{(t)} = \left[\left(\boldsymbol{x}_i^{(t)}\right)^T, \left(\boldsymbol{v}_i^{(t)}\right)^T\right]^T$, where $\boldsymbol{v}_i^{(t)}$ denotes its velocity at time t. The state is assumed to evolve over time according to

$$\boldsymbol{s}_i^{(t+1)} = f\left(\boldsymbol{s}_i^{(t)}, \boldsymbol{z}_i^{(t)}\right), \tag{7}$$

where $f(\cdot)$ is application-dependent, and $\boldsymbol{z}_i^{(t)}$ is the process noise. The proposed cooperative localization algorithm is then given in Algorithm 1.

Algorithm 1 Efficient cooperative positioning.

Initialization: For each tag i, initialize the state belief with $\left\{\boldsymbol{s}_i^{(0)}[n], w_i^{(0)}[n]\right\}_{n=1}^{\mathrm{N}}$, where $\boldsymbol{s}_i^{(0)}[n] = \left[\left(\boldsymbol{x}_i^{(0)}[n]\right)^T, \left(\boldsymbol{v}_i^{(0)}[n]\right)^T\right]^T$, $\boldsymbol{x}_i^{(0)}[n]$ and $\boldsymbol{v}_i^{(0)}[n]$ are generated according to the corresponding prior distributions, $w_i^{(0)}[n] = \frac{1}{\mathrm{N}}$.

Position Update:

For each time step t

For each mobile node i

1) Generate N samples $\left\{\boldsymbol{s}_i^{(t)}[n], w_i^{(t)}[n]\right\}_{n=1}^{\mathrm{N}}$ from $\left\{\boldsymbol{s}_i^{(t-1)}[n], w_i^{(t-1)}[n]\right\}_{n=1}^{\mathrm{N}}$ according to (7).
2) Measure distances to neighbouring nodes, and receive position/belief from them.
3) Compute the messages from neighbouring anchors and peer nodes using (4) and (6), respectively.
4) Update belief, set
$$w_i^{(\mathrm{t})}[n] = w_i^{(\mathrm{t})}[n] \prod_{j \in \Gamma_i^{(t)}} m_{i,j}\left(x_i^{(t)}[n]\right), \tag{8}$$
5) Estimate the mean $\boldsymbol{\mu}_i^{(t)}$ and covariance $\boldsymbol{\Sigma}_i^{(t)}$ of the belief by $\mu_i^{(t)} = \sum_{n=1}^{N} w_i^{(t)}[n]\boldsymbol{x}_i^{(t)}[n]$ and $\boldsymbol{\Sigma}_i^{(t)} = \frac{\sum_{n=1}^{N} w_i^{(t)}[n]\left(x_i^{(t)}[n] - \mu_i^{(t)}\right)\left(x_i^{(t)}[n] - \mu_i^{(t)}\right)^T}{1 - \sum_{n=1}^{N}\left(w_i^{(t)}[n]\right)^2}$.
6) Broadcast $\left(\boldsymbol{\mu}_i^{(t)}, \boldsymbol{\Sigma}_i^{(t)}\right)$ to the neighbours.
7) Resample $\left\{\boldsymbol{s}_i^{(t)}[n], w_i^{(t)}[n]\right\}_{n=1}^{\mathrm{N}}$ as in particle filters (Arulampalam *et al.*, 2002).

Table 1 compares the communication overhead and computational complexity of the proposed algorithm (assuming that (6) can be evaluated using a closed-form expression) with those of SPAWN and H-SPAWN. It can be seen that the communication and computational cost of the proposed algorithm is identical to those of H-SPAWN and significantly lower than those of SPAWN.

	Proposed scheme	H-SPAWN	SPAWN
Communication overhead[(a)]	9	9	4M
Computational complexity[(b)]	$O(N\|\Gamma\|)$	$O(N\|\Gamma\|)$	$O(NM\|\Gamma\|)$

(a) number of real values transmitted per node per time step.

(b) floating point operations (FLOPs) required per node per time step, where N and M denote the number of particles used locally and passed between nodes, respectively; $|\Gamma|$ is the number of neighboring mobile nodes.

Table 1. Comparison of communication overhead and computational complexity.

Since the ranging error distribution (i.e., $p(v)$ in (4) and (6)) is essential for both the accuracy and the efficiency of Algorithm 1, three different types of empirical ranging error models are proposed for NLOS environments in the following section. All of these models render closed-form expressions for (6), and system designers can choose one of them that best suits the application environment.

4. EMPIRICAL RANGING ERROR MODEL FOR NLOS PROPAGATION ENVIRONMENTS

To investigate the typical ranging error distributions in NLOS environments, an extensive ranging-data-collecting campaign was conducted in an indoor environment, using the wireless ad hoc system for positioning (WASP) developed at the Commonwealth Scientific and Industrial Research Organisation (CSIRO) of Australia. The WASP platform was designed with low-cost off-the-shelf hardware and operates in the 5.8 GHz ISM band. The system utilizes a bandwidth of 125 MHz for ranging measurements based on time-of-arrival (TOA). The WASP nodes form a multi-hop ad-hoc network during operation, which allows all the available ranging information between pairs of nodes to be collected. Detailed information about WASP system can be found in Sathyan *et al.* (2011) and references therein.

A WASP network containing 28 nodes at known coordinates was deployed in an office building, the floorplan of which is shown in Figure 2. It can be seen that the area contains many walls, resulting in predominantly NLOS conditions and substantial multipath reflections. The recorded ranging measurements were compared with the true locations of the nodes to obtain the ranging errors. Figure 3 shows the empirical ranging error distribution derived from around two million values. It can be seen that the distribution is strictly non-Gaussian, and exhibits a very long tail on the right hand side, which is caused by the multipath propagation of radio signals[2].

To characterize asymmetrical ranging error distributions as that in Figure 2, three types of ranging error models are proposed in the following, which render closed-form expressions for peer-to-peer message computation in (6).

[2] Compared with existing investigations into indoor ranging error distributions based on ultra-wideband (UWB) systems (see e.g., Wymeersch *et al.*, 2009, Alsindi *et al.*, 2009, Conti *et al.*, 2012), Figure 3 reflects the ranging error distributions in systems with narrower bandwidth.

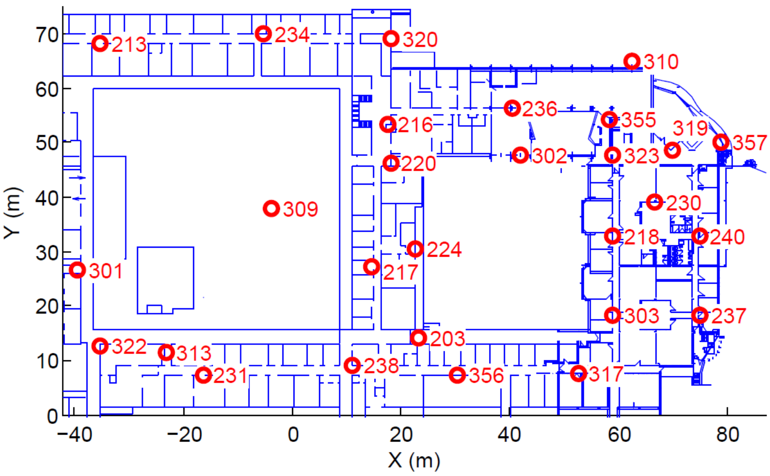

Figure 2. Floorplan of the deployed WASP network. The red circles denote the WASP nodes with known locations.

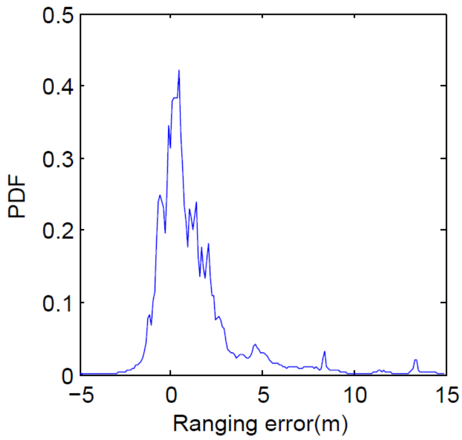

Figure 3. Empirical distribution of the recorded ranging errors.

4.1 Asymmetric Gaussian Ranging Error Model

One may consider using skew/asymmetric Gaussian distributions (Arellano-Valle *et al.*, 2005) to model asymmetrical ranging error distributions as shown in Figure 3. Specifically, the probability distribution function (PDF) of the ranging error is given by

$$p_{\mathrm{AG}}(v) = \begin{cases} \frac{2}{\sqrt{2\pi}(\sigma_{\mathrm{N}}+\sigma_{\mathrm{P}})} \mathrm{e}^{-\frac{v^2}{2\sigma_{\mathrm{N}}^2}} & v \leq 0 \\ \frac{2}{\sqrt{2\pi}(\sigma_{\mathrm{N}}+\sigma_{\mathrm{P}})} \mathrm{e}^{-\frac{v^2}{2\sigma_{\mathrm{P}}^2}} & v \geq 0 \end{cases}, \quad (9)$$

where σ_{N}^2 and σ_{P}^2 are the scale parameters corresponding to the positive part and negative part of the distribution (note that (9) reduces to a Gaussian distribution when $\sigma_N = \sigma_P$).

Substituting (9) for $p(v)$ in (6), it can be shown that the peer-to-peer messages can be computed using a closed-form expression as below:

$$m_{i,j}(\boldsymbol{x}_i) \propto \frac{1}{\pi\sigma_j(\sigma_P+\sigma_N)} \left\{ e^{-\frac{d^2}{2\left(\sigma_j^2+\sigma_P^2\right)}} \sqrt{\frac{\pi}{2}} \frac{\sigma_j\sigma_P}{\sqrt{\sigma_j^2+\sigma_P^2}} erfc\left(-\frac{\sigma_P d}{\sqrt{2}\sigma_j\sqrt{\sigma_j^2+\sigma_P^2}} \right) + \right. \quad (10)$$

$$e^{-\frac{d^2}{2\left(\sigma_j^2+\sigma_N^2\right)}}\left[\sqrt{2\pi}\frac{\sigma_j\sigma_N}{\sqrt{\sigma_j^2+\sigma_N^2}}-\sqrt{\frac{\pi}{2}}\frac{\sigma_j\sigma_N}{\sqrt{\sigma_j^2+\sigma_N^2}}erfc\left(-\frac{\sigma_N d}{\sqrt{2}\sigma_j\sqrt{\sigma_j^2+\sigma_N^2}}\right)\right]\Bigg\},$$

where $d = r_{i,j} - \left\|\boldsymbol{x}_i - \boldsymbol{\mu}_j\right\|_2$, $erfc(\cdot)$ denotes the complementary error function.

4.2 Asymmetric Double Exponential Ranging Error Model

To characterize the long tails of the ranging error distribution as shown in Figure 3, one can use an asymmetric double exponential model defined as follows:

$$p_{\mathrm{ADE}}(v) = \begin{cases} \frac{1}{\lambda_{\mathrm{P}}+\lambda_{\mathrm{N}}} e^{\frac{v}{\lambda_{\mathrm{N}}}} & v \le 0 \\ \frac{1}{\lambda_{\mathrm{P}}+\lambda_{\mathrm{N}}} e^{-\frac{v}{\lambda_{\mathrm{P}}}} & v \ge 0 \end{cases}, \quad (11)$$

where λ_{P} and λ_{N} are the scale parameters. Substituting (16) for $p(v)$ in (6) and computing the resulting integration also gives an analytical expression for peer-to-peer message computation:

$$m_{i,j}(\boldsymbol{x}_i) \propto \frac{1}{\lambda_{\mathrm{P}}+\lambda_{\mathrm{N}}}\left\{Q\left(-\frac{d}{\sigma_j}+\frac{\sigma_j}{\lambda_{\mathrm{P}}}\right)e^{-\frac{d}{\lambda_{\mathrm{P}}}+\frac{\sigma_j^2}{2\lambda_{\mathrm{P}}^2}}+\left[1-Q\left(-\frac{d}{\sigma_j}-\frac{\sigma_j}{\lambda_{\mathrm{N}}}\right)\right]e^{\frac{d}{\lambda_N}+\frac{\sigma_j^2}{2\lambda_{\mathrm{N}}^2}}\right\}, \quad (12)$$

where $\mathrm{Q}(\cdot)$ denotes the Q-function, $d = r_{i,j} - \left\|x_i - \mu_j\right\|_2$.

4.3 Shifted Rayleigh Ranging Error Model

Another analytical probability distribution that can also be used to model asymmetrical ranging error distributions is the shifted Rayleigh distribution, which is given as:

$$p_{\mathrm{SR}}(v) = \begin{cases} 0 & v \le \mu_{\mathrm{SR}} \\ \frac{v-\mu_{\mathrm{SR}}}{\sigma_{\mathrm{SR}}^2} e^{-\frac{(v-\mu_{\mathrm{SR}})^2}{2\sigma_{\mathrm{SR}}^2}} & v \ge \mu_{\mathrm{SR}} \end{cases}, \quad (13)$$

The corresponding equation for computing peer-to-peer messages is given by

$$m_{i,j}(\boldsymbol{x}_i) = \frac{\sigma_{\mathrm{SR}} d}{\left(\sigma_j^2+\sigma_{\mathrm{SR}}^2\right)^{\frac{3}{2}}} \mathrm{e}^{-\frac{d^2}{2\left(\sigma_j^2+\sigma_{\mathrm{SR}}^2\right)}} Q\left(\frac{\sigma_{\mathrm{SR}}}{\sigma_j}\frac{d}{\sqrt{\sigma_j^2+\sigma_{\mathrm{SR}}^2}}\right) + \frac{\sigma_j}{\sqrt{2\pi\left(\sigma_j^2+\sigma_{\mathrm{SR}}^2\right)}} \mathrm{e}^{-\frac{d^2}{2\sigma_j^2}}, \quad (14)$$

where $d = r_{i,j} - \left\|\boldsymbol{x}_i - \boldsymbol{\mu}_j\right\|_2 - \mu_{\mathrm{SR}}$.

The proposed empirical ranging error models are summarized in Table 2. To use these models in practice, the following steps should be followed during the deployment of the system (note that the first two steps are conducted offline, before putting the system into operation):

1) Collect sample ranging error data in the application environment, using the anchor nodes or specifically/temporally deployed nodes at known locations.
2) Fit the recorded ranging errors to the ranging error models and choose the one that achieves the best fit (see APPENDIX B for the method of parameter estimation).
3) Use the corresponding equations for message computation in Algorithm 1.

Note that in addition to the models listed in Table 2, it is believed there are other analytical functions that can be used to model ranging error distribution and render closed-form expressions for (6), which is of interest in future studies.

Model Name	Asymmetric Gaussian	Asymmetric Double Exponential	Shifted Rayleigh
Parameters	σ_P, σ_N	λ_P, λ_N	μ, σ
PDF	(9)	(11)	(13)
Equation for peer-to-peer messages computation	(10)	(12)	(14)
Parameter estimation for model fitting	(19)(20)	(21)(22)	(23)(24)

Table 2. Summary of the proposed ranging error models.

To demonstrate the usage of the proposed ranging error models, the recorded ranging data as described above was fitted to the three models, with the results shown visually in Figure 4. It can be seen that the asymmetric double exponential model gives the best fit for the experimental environment under consideration. The performance of using this model in Algorithm I will be investigated experimentally in the following section.

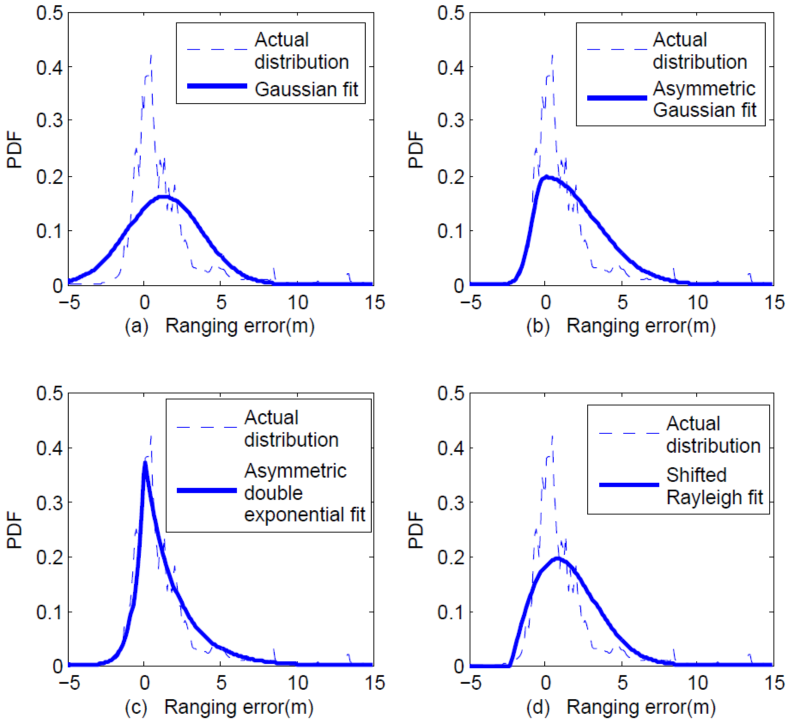

Figure 4. Fitting the recorded ranging error to the proposed ranging error models. (a): Gaussian model ($\mu = 1.33$, $\sigma^2 = 6.08$); (b): Asymmetric Gaussian model ($\sigma_P^2 = 10.46$, $\sigma_N^2 = 0.65$); (c): Asymmetric double Exponential model ($\lambda_P = 2.03$, $\lambda_N = 0.60$); (d): Shifted Rayleigh model ($\mu = -2.17$, $\sigma^2 = 9.20$).

5. EXPERIMENTAL RESULTS

The performance of the proposed scheme was evaluated experimentally on the WASP system, using an experiment setup similar to that described in Section 4. Specifically, a WASP network containing 28 static nodes and one mobile node was deployed, as is shown in Figure 6. The four WASP nodes marked by red squares were used as anchors, while the other 24 static nodes marked by red circles and the mobile node were treated as tags. The nearly constant velocity (NCV) model (Sathyan and Hedley, 2013, Bar-Shalom Y and Li XR, 2001) was used to model the state evolution (cf. (7)) of tags (including the 24 static tags, it is assumed the algorithm has no knowledge on whether the tags are static or not).

The following algorithms were employed to locate the tags:

- **Proposed**: the proposed Algorithm 1, using the asymmetric double exponential ranging error model in Figure 4 (c).
- **SPAWN-N**: SPAWN of Wymeersch *et al.* (2009), where beliefs exchanged between nodes were represented by M weighted particles. M was chosen to be equal to N (the number of particles used locally within each tag). The asymmetric double exponential ranging error model was used.
- **SPAWN-5**: SPAWN with M = 5, which exhibits a complexity in the same order as that of the proposed algorithm[3].
- **H-SPAWN**: H-SPAWN of Caceres *et al.* (2011), using the Gaussian ranging error model as shown in Figure 4 (a).

Figure 5 shows the cumulative distribution of positioning errors corresponding to the static tags. The number of particles used locally (N) was 500 for all the considered algorithms. It is shown that the performance of the proposed algorithm is very close to that of SPAWN-N (note once again that SPAWN-N is close to optimal and can be used as an upper bound for performance comparison). In addition, 80 percent of the position errors are below 1.2 m for the proposed algorithm, and 90 percent of the errors are below 1.7 m, which outperforms SPAWN-5 and H-SPAWN. Note that the corresponding accuracies for H-SPAWN are 2.6 m and 3.3 m respectively, which is due to the mismatch in ranging error model (see Figure 4).

The estimated trajectory of the moving tag in one of the experiments was shown in Figure 6. It can be seen that the trajectory conforms to the corridors in the building most of the time, which indicates that the position of the moving node is tracked accurately. The estimated path in the upper-left region of the figure is less accurate than those in other regions, which is due to the lower node density in that area.

In order to evaluate the positioning accuracy of the moving tag quantitatively, one can take advantage of the fact that the WASP node moved along the centre of corridors and compute the deviation of each estimated positions from the centre line of the corresponding corridors (position deviation). Figure 7 shows the cumulative distribution of the position deviations in the right hand side of the experimental area (note that SPAWN-N was not included as its very high computational complexity made it infeasible to compute a complete tracking solution). It can be seen that the fraction of position deviations less than 1 m is 95 percent using the proposed algorithm, which is substantially higher than those achieved by SPAWN-5 (80%) and H-SPAWN (72%). The RMSE of position deviation for the proposed algorithm is 0.5 m, which corresponds to a RMSE position error of 0.7 m if the positioning error distribution is isotropic.

An interesting phenomenon in Figure 5 and Figure 7 is that although the beliefs passed between neighbours are represented by only five particles, the performance of SPAWN-5 is higher than that of H-SPAWN. This is mainly because SPAWN-5 uses a more suitable

[3] Note that SPAWN is close to optimal given sufficient communication bandwidth and computational capability. This baseline scheme is included to compare the proposed algorithm with particle-based SPAWN under similar constraints of complexity.

ranging error model than HSPAWN does. In addition, the fact that the beliefs of the tags are stationary (for the static tags) or correlated (for the mobile tag) over time allows SPAWN-5 to converge after a sufficient number of time steps, which balances out the shortcomings of using only five particles. However, it should be noted that the proposed algorithm outperforms SPAWN-5 with significantly lower communication and computational cost.

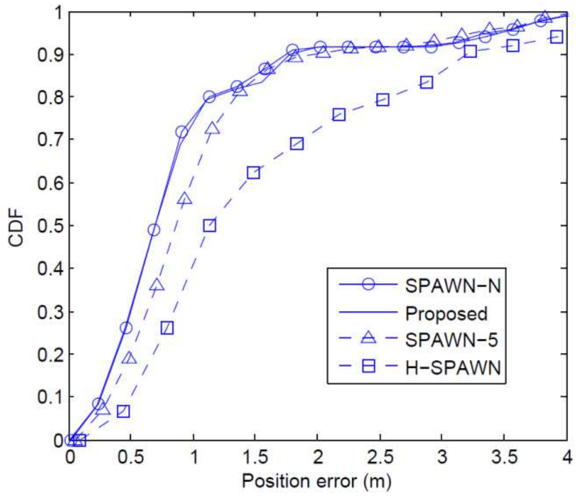

Figure 5. Cumulative distribution of positioning errors for static tags.

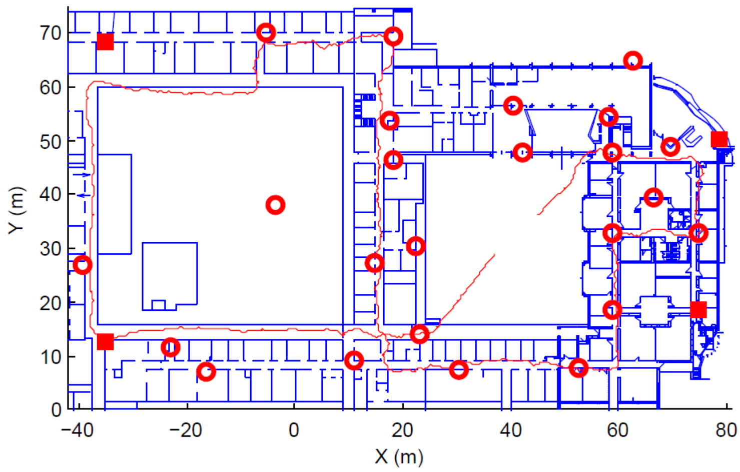

Figure 6. Estimated trajectory of the moving node. Red squares and red circles denote the 28 static WASP nodes, with those marked by red squares treated as anchors.

6. CONCLUSIONS

A framework for efficient cooperative localization is proposed based on distributed belief propagation. Beliefs passed between neighbours are approximated by Gaussian distributions, and an approximation is proposed to compute peer-to-peer messages efficiently. In addition, multiple empirical ranging error models are proposed, which can be used to characterize ranging error distributions under NLOS conditions and render closed-form expressions for belief updating in the proposed cooperative localization algorithm. The proposed scheme is more efficient and general than the existing approaches. Experimental results in a deployed indoor localization system show that the algorithm achieves high accuracy with low communication overhead and computational complexity.

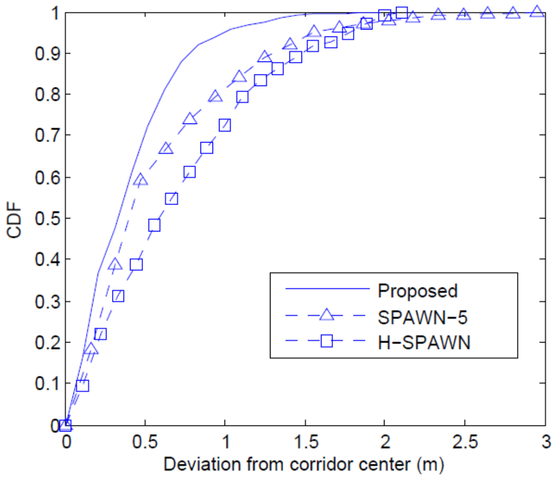

Figure 7. Cumulative distribution of the deviation of estimated positions from corridor centres.

APPENDIX A – MARGINAL VARIANCE OF A 3-D GAUSSIAN DISTRIBUTION ALONG A LINE

To obtain the marginal variance of a 3-D Gaussian distribution along a line, one can firstly rotate the coordinate system so that the X-axis of the new coordinate system aligns with the line. The marginal variance is then obtained by taking the (1,1)-th element of the resulting covariance matrix represented in the resulting coordinate system. Specifying the direction of the line by polar angle ϕ and azimuthal angle θ, one can firstly rotate the initial coordinate system about the Z-axis by θ, and then about the Y-axis by $\phi - \frac{\pi}{2}$. The resulting covariance matrix in the new coordinate system is given by:

$$\boldsymbol{\Sigma}_j' = \boldsymbol{C}_2^T \boldsymbol{C}_1^T \boldsymbol{\Sigma}_j \boldsymbol{C}_1 \boldsymbol{C}_2, \tag{15}$$

where

$$\boldsymbol{C}_1 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

$$\boldsymbol{C}_2 = \begin{bmatrix} \sin(\phi) & 0 & -\sin(\phi) \\ 0 & 1 & 0 \\ \cos(\phi) & 0 & \cos(\phi) \end{bmatrix}. \tag{17}$$

It can then be shown that the marginal variance is given by

$$\sigma^2 = \boldsymbol{u}^T \boldsymbol{\Sigma}_j \boldsymbol{\mu}, \tag{18}$$

where $\boldsymbol{\mu} = [\sin(\phi)\cos(\theta) \quad \sin(\phi)\sin(\theta) \quad \cos(\phi)]^{\mathrm{T}}$.

APPENDIX B – MAXIMUM LIKELIHOOD ESTIMATION OF PARAMETERS FOR THE PROPOSED RANGING ERROR MODEL

Denoting the ranging errors recorded during surveying campaign as $\{v_i\}_{i=1}^K$, the parameters of the ranging error models can be estimated according to the criteria of maximum likelihood.

For asymmetric Gaussian model, σ_{P}^2, σ_{N}^2 can be estimated by

$$\sigma_{\mathrm{P}}^2 = \frac{1}{\sum_{i=1,v_i\geq 0}^{K} 1} \sum_{i=1,v_i\geq 0}^{K} v_i^2, \tag{19}$$

and

$$\sigma_{\mathrm{N}}^2 = \frac{1}{\sum_{i=1,v_i\leq 0}^{K} 1} \sum_{i=1,v_i\leq 0}^{K} v_i^2, \tag{20}$$

respectively, where $\sum_{i=1,v_i\geq 0}^{K} 1$ and $\sum_{i=1,v_i\leq 0}^{K} 1$ denote the number of non-negative and non-positive ranging errors, respectively.

For asymmetric double exponential model, λ_{P} and λ_{N} can be estimated according to

$$\lambda_{\mathrm{P}} = \frac{1}{\sum_{i=1,v_i\geq 0}^{K} 1} \sum_{i=1,v_i\geq 0}^{K} v_i, \tag{21}$$

and

$$\lambda_{\mathrm{N}} = \frac{-1}{\sum_{i=1,v_i\leq 0}^{K} 1} \sum_{i=1,v_i\leq 0}^{K} v_i, \tag{22}$$

respectively.

Since the PDF of a shifted Rayleigh distribution is zero if $v \leq \mu_{\mathrm{SR}}$ (see (13)), only those v_i with values greater than μ_{SR} can be used for model fitting. Since μ_{SR} is one of the unknown parameters that need to be estimated, there is no closed-form expressions for fitting the ranging errors to a shifted Rayleigh model. However, a heuristic method can be employed to estimate μ_{SR} by numerically maximizing the log-likelihood function, i.e.,

$$\mu_{\mathrm{SR}} = \max_{\mu_{\mathrm{SR}}} \frac{K}{\sum_{i=1,v_i>\mu_{\mathrm{SR}}}^{K} 1} \sum_{i=1,v_i>\mu_{\mathrm{SR}}}^{K} \log\left[\frac{1}{\sigma^2} e^{-\frac{(v_i-\mu_{\mathrm{SR}})^2}{2\sigma^2}}\right], \tag{23}$$

where $\frac{K}{\sum_{i=1,v_i\geq\mu_{\mathrm{SR}}}^{K} 1}$ imposes a penalty for discarding those v_i less than or equal to μ_{SR}, σ^2 is obtained by fitting $v_i - \mu_{\mathrm{SR}}$ to a Rayleigh distribution and is given by

$$\sigma^2 = \frac{1}{2\sum_{i=1,v_i>\mu_{\mathrm{SR}}}^{K} 1} \sum_{i=1,v_i>\mu_{\mathrm{SR}}}^{K} (v_i - \mu_{\mathrm{SR}})^2. \tag{24}$$

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REFERENCES

Alsindi N, Alavi B, Pahlavan K (2009) Measurement and modeling of ultra-wideband TOA-based ranging in indoor multipath environments, IEEE Transactions on Vehicular Technology 58(3): 1046–1058

Arellano-Valle RB, Gomez HW, Quintana FA (2005) Statistical inference for a general class of asymmetric distributions, Journal of Statistical Planning and Inference 128(2): 427–443

Arulampalam M, Maskell S, Gordon N, Clapp T (2002) A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking, IEEE Transactions on Signal Processing 50(2): 174–188

Bar-Shalom Y, Li XR (2001) Estimation with applications to tracking and navigation, John Wiley & Sons, Inc., New York, NY, USA

Biswas P, Liang TC, Toh KC, Ye Y, Wang TC (2006) Semidefinite programming approaches for sensor network localization with noisy distance measurements, IEEE Transactions on Automation Science and Engineering 3(4): 360–371

Caceres M, Penna F, Wymeersch H, Garello R (2011) Hybrid cooperative positioning based on distributed belief propagation, IEEE Journal on Selected Areas in Communications 29(10): 1948–1958

Conti A, Guerra M, Dardari D, Decarli N, Win M (2012) Network experimentation for cooperative localization, IEEE Journal on Selected Areas in Communications 30(2): 467–475

Costa JA, Patwari N, Hero AO (2006) Distributed weighted-multidimensional scaling for node localization in sensor networks, ACM Transactions on Sensor Networks 2(1): 39–64

Ihler A, Fisher J, Moses R, Willsky A (2005) Nonparametric belief propagation for self-localization of sensor networks, IEEE Journal on Selected Areas in Communications 23(4): 809–819

Lien J, Ferner UJ, Srichavengsup W, Wymeersch H, Win M (2012) A comparison of parametric and sample-based message representation in cooperative localization, International Journal of Navigation and Observation 2012

Meyer F, Hlawatsch F, Wymeersch H (2013) Cooperative simultaneous localization and tracking (coslat) with reduced complexity and communication, IEEE International Conference on Acoustics, Speech and Signal Processing 2013: 4484–4488

Meyer F, Hlinka O, Hlawatsch F (2014) Sigma point belief propagation, IEEE Signal Processing Letters 21(2): 145–149

Patwari N, Ash JN, Kyperountas S, Hero AO, Moses RL, Correal NS (2005) Locating the nodes: Cooperative localization in wireless sensor networks, IEEE Signal Processing Magazine 22(4): 54-69

Sathyan T, Hedley M (2013) Fast and accurate cooperative tracking in wireless networks, IEEE Transactions on Mobile Computing 12(9): 1801–1813

Sathyan T, Humphrey D, Hedley M (2011) WASP: A system and algorithms for accurate radio localization using low-cost hardware, IEEE Transactions on Systems, Man and Cybernetics 41(2): 211–222

Savic V, Zazo S (2012) Reducing communication overhead for cooperative localization using nonparametric belief propagation, IEEE Wireless Communications Letters, 1(4): 308–311

Shen Y, Wymeersch H, Win M (2010) Fundamental limits of wideband localization part II: cooperative networks," IEEE Transaction on Information Theory 56(10): 4981–5000

Sottile F, Wymeersch H, Caceres M, Spirito M (2011) Hybrid GNSS-terrestrial cooperative positioning based on particle filter, Proceedings of IEEE Global Communications Conference 2011

Tseng P (2007) Second-order cone programming relaxation of sensor network localization, SIAM Journal on Optimization 18(1): 156–185

Wymeersch H, Lien J, Win M (2009) Cooperative localization in wireless networks, Proceedings of the IEEE, 97(2): pp. 427–450

Xiong Z, Sottile F, Caceres M, Spirito M, Garello R (2011) Hybrid WSN-RFID cooperative positioning based on extended Kalman filter, IEEE-APS Topical Conference on Antennas and Propagation in Wireless Communications (APWC) 2011: 990–993