International Global Navigation Satellite Systems Society
IGNSS Symposium 2007

The University of New South Wales, Sydney, Australia
4 – 6 December, 2007

Adaptive Calibration of a Magnetometer Compass for a Personal Navigation System

Shahram Moafipoor
Satellite Positioning and Inertial Navigation (SPIN) Laboratory
Center for Mapping
The Ohio State University
Tel. +1-614-292-4950, Fax. +1-614-292-8062
shahram@cfm.ohio-state.edu

Dorota A. Grejner-Brzezinska
Department of Civil and Environmental Engineering and Geodetic Science
Satellite Positioning and Inertial Navigation (SPIN) Laboratory
The Ohio State University
Tel. +1-614-292-8787, Fax. +1-614-292-2957
dbrzezinska@osu.edu

Charles Toth
Center for Mapping
Satellite Positioning and Inertial Navigation (SPIN) Laboratory
The Ohio State University
Tel. +1-614-292-7681, Fax. +1-614-292-8062
toth@cfm.ohio-state.edu

ABSTRACT

This paper discusses experiences in estimating the heading angle from a 3-axis Honeywell HMR3000 magnetometer compass. The magnetometers are used for absolute heading determination with reference to local magnetic north, where the heading is derived from the horizontal force of the magnetic field. However, any ferromagnetic materials near the sensor superimpose extra magnetic fields on the local Earth's magnetic field. This disturbance could affect the local magnetic field intensity and eventually the heading measured by the magnetometer. To calibrate the HMR3000, an autocalibration procedure was first implemented (without a need for any external reference directions) to compensate for the local variation of the Earth's magnetic field. The autocalibration procedure was intentionally designed to (1) estimate the magnetic field effects, (2) compensate for these effects in the magnetometer measurements in terms of bias and scale factor for each axis, (3) compensate the tilt angle, and (4) apply the magnetic declination angle.

In this paper, a multilayer perceptron artificial neural network (ANN) for dynamic calibration is proposed and implemented. The inputs to the network are (1) raw heading calculated from the uncalibrated magnetometer, (2) total magnetic flux density, (3) gyro heading, (4) the vertical component of the gravity vector (inclination angle) and (5) temperature. The network is trained based on the true heading measured either by the GPS velocity (in case of accessible GPS), rate gyro (in case of calibrated gyro), or map-direction constraint (in case of Dead Reckoning supported by map data).

The proposed algorithms, autocalibration and dynamic calibration based on the neural network, have been tested on real data. The numerical results show the accuracy better than 4-6º in both autocalibration and dynamic calibration for heading estimation. These results suggest that the proposed algorithms can provide reliable heading measurements from the HMR3000 magnetometer compass.

KEYWORDS: Personal navigation, magnetometer compass, calibration, neural networks.

1. INRODUCTION

The personal navigator developed at The Ohio State University includes a component to provide navigation information based on a Dead Reckoning (DR) algorithm used when the GPS signals are not available. DR is a relative measurement approach, the fundamental idea of which is to integrate incremental motion information over time. Starting from a known position before GPS outage, successive position displacements are accumulated. The displacement estimates are typically in the form of changes in step length and heading of the operator. Step length estimation was reported previously in Grejner-Brzezinska *et al.*, 2007; Moafipoor *et al.*, 2007. In this paper, however the main focus is restricted to heading determination.

Heading in the OSU personal navigator is obtained from the HMR3000 magnetometer compass and the HG1700 IMU. The HG1700 IMU with gyro stability better than 3-5 º/hr, is sufficiently reliable to provide heading for a limited time during GPS outages, as after a limited time, the size of the gyro drift necessitates recalibration. Furthermore, the integration of gyro rates requires an initial heading value (Jekeli, 2000). The HMR3000 magnetometer compass, on the other hand, measures absolute orientation through a combination of three-axis magnetic sensors and two-axis tilt sensors. The magnetic compass senses the Earth's magnetic field intensity (Hoff and Azuma, 2000), and the tilt sensor (inclinometer) senses the direction of the gravity. If the magnetometer were aligned with the local horizontal plane, the heading, α, would be calculated as Eq. 1, using the sign of the argument to determine the quadrant of the value.

$$\alpha = \tan^{-1}\left(\frac{H_y}{H_x}\right) \tag{1}$$

where, H_x, H_y represent the horizontal magnetic field components of the Earth. The Earth's magnetic field is a weak signal with a range from 0.5 to 0.65 gauss, or 50,000 nT (milligauss=100 nT). The precision of a magnetometer compass depends upon the precision in magnetic and tilt measurements. The HMR3000 with a MagnetoResistive (MR) magnetic sensor has a sensor resolution better than 0.1 milligauss, with respond time less than 0.1 micro-second, up to 20Hz read-out rate, 1º (level) and 2º (tilt) heading accuracy (1 sigma). Ideally, the magnetometer must be in its horizontal position to determine the heading direction accurately, as tilting the sensor will reduce its accuracy (Ladetto and Marminod, 2002). The measured magnetic azimuth is also subject to magnetic disturbances, superimposed into the Earth's magnetic field (Caruso, 2000). All these facts indicate that the quality of the magnetometer heading strongly depends on tilt compensation and the calibration procedure to compensate for the local variations of the Earth's magnetic field, including identifying possible error sources and removing them from the measurements. To calibrate the HMR3000, two different approaches can be considered, one requires no external reference heading (auto-calibration), while another one is facilitated by using the reference heading coming from, for example, GPS velocity (in the case of accessible GPS), rate gyro (in the case of a calibrated gyro) or map-direction constraint.

First, an autocalibration procedure to compensate for the local variation of the Earth's magnetic field was implemented. This autocalibration procedure, inspired by Caruso (1997), is based on the fact that the locus of error-free magnetometer measurements is a circle if the sensor moves around a loop. The impact of various magnetometer errors would distort the shape of this circle; however, the circular constraint eventually can be used to partially estimate local variations of the Earth's magnetic field. A practical implementation of the autocalibration does not require any reference headings, but this method strongly depends on position and the effect of ferromagnetic materials in the proximity of the sensor. If the sensor's location is changed and it is exposed to a new environment, a new calibration is necessary for identifying and removing effects of the new magnetic environment.

The second approach is based on using external reference heading. In this method, the difference between the true heading and compass heading, also known as compass deviation, is fed back to the magnetometer to calibrate it. Liu *et al*. (1989) developed a mathematical model for compass deviation in the form of a polynomial with four unknown coefficients. Once these coefficients are estimated, they create an a priori look-up table from which heading corrections are interpolated. New studies show that the compass deviation model is more complicated than a linear model, with additional coefficients that are a function of the local Earth's magnetic field. Subsequently, a Kalman filtering approach was investigated by Hoff and Azuma (2000). The goal of the approach was to estimate the angular rotation from the input of a magnetometer compass and three gyroscopes. The idea is based on using the differential equations integrated of rate gyro to obtain heading angle as a function of time. Another approach is based on the least squares estimation of the compass deviation model (Gebre-Egziabher *et al*., 2001); this method was later improved into a constrained total least squares technique (Tsai, 2006). The objective behind these methods is to approximate the compass deviation model from a non-linear to a linear model, and then estimate its corrections.

In this paper, the non-linear compass deviation model is approximated using an Artificial Neural Network (ANN). Wang and Gao (2006) have demonstrated the capability of such a system to calibrate the compass measurements. When an external heading reference is available, ANN is trained to model the non-linear relationship between the true heading and measured heading. After training, the ANN could convert uncalibrated compass heading into the corrected heading. For an outdoor experiment, precision of around 5-7º was reported. To improve the accuracy, this paper tries to strengthen the ANN performance in terms of structure, input and training data, to obtain a better accuracy for indoor and outdoor experiments. The reminder of the paper is organized as follows. Section 2 explores the error analysis of the magnetometer compass; Section 3 discusses the ANN design and its contribution to estimation the heading parameter; Sections 4 and 5 illustrate the experimental results for the proposed autocalibration procedure and ANN approach. Finally, the summary and conclusion are presented in Section 6.

2. MAGNETOMETER MEASUREMENT ERROR ANALYSIS

A comprehensive study on possible magnetometer error sources reveals five significant factors which can impact compass accuracy: (1) magnetic sensor sensitivity; (2) nearby ferrous materials; (3) sensor tilt; (4) declination angle, and (5) temperature.

- Magnetic sensor sensitivity

The magnetic sensor error explains the sensitivity of the magnetic sensor response. The magnetic fields in the horizontal plane are typically in the range of 200-300 milligauss. For a 0.1º resolution, an equivalent magnetic sensitivity of better than 0.35 milligauss is necessary (Caruso, 2000). However, because of instrumentation errors, the level of magnetometer sensitivity varies from one axis to the next. Therefore, it is necessary to define three individual scale factors for each axis.

- Nearby ferrous materials

When the magnetometer is close to ferrous materials, it measures the Earth's magnetic field corrupted by the local magnetic field, coming from these materials. The ferrous materials are usually classified into two groups, hard iron and soft iron. The magnetic field of the hard iron remains constant in a fixed location relative to the compass for all heading directions (Kao and Tsai, 2006). As a result, it adds a constant magnetic field component to the Earths' magnetic field. This distortion can be determined systematically and applied to subsequent output readings. The soft iron materials, on the other hand, condense magnetic flux toward themselves, causing distortion depending on the compass direction (Gouws and Merwe, 2000); for example, power cables can generate strong magnetic fields. The hard and soft iron effects can distort the compass heading to more than tens of degree (Caruso, 2000). Combining the effect of hard and soft irons with the magnetic sensor sensitivity results in measurement magnetic error given by:

$$\hat{\vec{H}}_b = \vec{H}_b + S_{SF} \times \vec{H}_b + \vec{B}_b \tag{2}$$

where $\hat{\vec{H}}_b$ is the measured magnetic field of the sensor, $\vec{H}_b$ is the true-output (unknown), S_{SF} is the induced magnetism coefficients matrix (unknown), and $\vec{B}_b$ is the permanent magnetism offset (unknown). In order to get the true output from the distorted measurements, Eq.2 can be rearranged as:

$$\vec{H}_b = (I_{3\times 3} + S_{SF})^{-1}(\hat{\vec{H}}_b - \vec{B}_b) \tag{3}$$

From Eq.3, it can be concluded that the locus of measurements under the distorted magnetic field is an ellipsoid, with the center at the permanent magnetism offset (Hu *et al.*, 2005).

- Sensor tilt

The magnetic heading is measured as the angle in the local horizontal plane measured clockwise from magnetic North. Therefore, the magnetometer must be horizontal to determine direction accurately; as tilting the sensor will reduce accuracy. The roll angle, θ, and the pitch angle, φ, delivered by the two-axis tilt sensor, are used to rotate the measured magnetic components to the horizontal as:

$$\begin{bmatrix} H_x^{Level} \\ H_y^{Level} \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\theta\sin\varphi & -\cos\theta\sin\varphi \\ 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}_b \tag{4}$$

where, H_x^{Level}, H_y^{Level} are the leveled magnetic components obtained. The simulation study done by Bourgeois and Martinez (1999) showed that for example, a pitch error of 0.3º and no roll error can contribute a 0.5º error on the heading.

- Declination angle

The declination angle, i.e. the depart angle between geographic and magnetic north, depends on the location of the sensor and can be determined from existent maps and tables at (http://www.ngdc.noaa.gov/seg/geomag/declination.shtml). The absolute error derived by this angle is defined as the difference between the compass reading and the real north (geographic north) direction, which should be added to the compass heading. For example, in the Columbus, Ohio, USA area, the declination angle is about -6.5º.

- Temperature

The sensitivity of the magnetometer, magnetic offset and resistance sensitivity change with temperature. This change will affect the measurements of the magnetic fields. The temperature causes two kinds of errors in measuring the magnetic field: an offset drift and a sensitivity parameter, called tempco (temperature coefficient). The maximum offset drift may reach up to 0.3º, even after precise manufacturer calibration, while the sensitivity tempco appears as a change in the output gain of the sensor over temperature (small effect) (Caruso, 1997). It also is shown by Wang *et al.* (2006) that the heading error has a non-linear relationship with temperature and measured angles.

To formulate the total instrumentation error, Eqs. (2) and (4) can be added to give:

$$\hat{\vec{H}}_b = S_{SF} C_{Soft-iron} (R_{\theta,\varphi} \vec{H}_b + \vec{B}_{hard-iron}) \tag{5}$$

where, $\hat{\vec{H}}_b$ represents the measured magnetic field, S_{SF} is a diagonal matrix with scale factors of the three magnetometer axes, $C_{soft-iron}$ is the matrix representing the induced effect of the soft iron with 9 unknown coefficients, $R_{\theta,\varphi}$ is the rotation angle to compensate for the tilt effect, as shown in Eq.4, $\vec{H}_b$ represents the true magnetic field in the body frame, and $\vec{B}_{hard-iron}$ is the permanent magnetic field superimposed on the output of the magnetometer measurements. If all of the error components in Eq.5 are estimated and applied, then heading can be determined by Eq.1 followed by adding the declination angle.

To estimate the unknown parameters, it is usual to approximate the Eq.5 by assuming the induced effect as the identity matrix, and applying the sensor tilt to the magnetometer measurements for only horizontal components, such that:

$$\begin{aligned} H_x &= S_x H_x^{Level} + B_x \\ H_y &= S_y H_y^{Level} + B_y \end{aligned} \tag{6}$$

where, S_x, S_y are two scale factors, and B_x, B_y are two biases along the horizontal axes of the magnetic field. To calibrate the magnetometer using Eq.6, a simple method is to perform the loop maneuver discussed earlier, and to determine scale factors as ratios of the major and minor axes, changing the circle to an ellipsoid, and bias parameters as the offset center of the ellipsoid. A practical implementation of the autocalibration does not require any external reference headings to compensate the local variation of the earth's magnetic field (permanent and induced magnetisms). The method proposed by Caruso (1997) is used to estimate the bias and scale factor parameters:

$$\begin{aligned} S_x &= Max\left(1, \frac{H_y^{Max} - H_y^{Min}}{H_x^{Max} - H_x^{Min}}\right), \quad S_y = Max\left(1, \frac{H_x^{Max} - H_x^{Min}}{H_y^{Max} - H_y^{Min}}\right) \\ B_x &= \left(\frac{H_x^{Max} - H_x^{Min}}{2} - H_x^{Max}\right) S_x, \quad B_y = \left(\frac{H_y^{Max} - H_y^{Min}}{2} - H_y^{Max}\right) S_x \end{aligned} \tag{7}$$

where H_x^{Max}, H_y^{Max}, H_x^{Min}, and H_y^{Min}, are the maximum and minimum of the measured magnetic field along x and y axes, respectively.

Another approach to estimate the magnetometer calibration parameters is based on using external reference directions. Substituting Eq.6 into the Eq.1 gives:

$$\alpha = \tan^{-1}\left(\frac{S_y H_y^{Level} + B_y}{S_x H_x^{Level} + B_x}\right) \tag{8}$$

The difference between the true heading and compass heading can be used to adjust the magnetometer parameters by a least squares technique. Because of the sensitivity of the model, the process requires good initial values; otherwise, the computation could easily lead to divergence (Gebre-Egziabher *et al.*, 2001). Haupt *et al.* (1996) proposed a two-step estimator where the first step is solved by using a standard batch least squares linear estimation technique and the second step is solved algebraically. Tsai (2006) proposed using constrained total least squares technique to solve such a non-linear parameter estimation, but for a completely different application. This is a promising idea for non-linear modeling of heading. The mathematical model and details of the total least squares adjustment with constraints can be found in, for example, Schaffrin and Felus (2005). The approach proposed here is to evaluate the magnetometer calibration parameters and finally to derive compass heading based on an ANN.

3. NEURAL NETWORK AND HEADING MODELING

The ANN approach was selected because the error sources of a magnetometer compass including sensor sensitivity, local magnetic disturbance superimposed on the Earth's magnetic field flux density, platform-induced magnetic, and magnetic declination anomaly are variables of a non-linear model. Furthermore, some of the magnetic anomalies cannot be parameterized by a parametric model, such as temperature.

ANNs are trainable dynamic systems that offer several capabilities, such as non-linear input-output mapping, generalization, adaptivity, and fault tolerance that stem from the interconnected structures and distributed data representation. ANNs are able to directly learn from training data and can generalize to new input data, which are different from the training data. Basically, ANNs can automatically adjust the connection weights to optimize their performance. In addition, a large number of connections provides sufficient redundancy against noisy input data. The performance of ANNs depends on the activation function of neurons, structure and topology of the network, learning method and representation of training and inputs to the network.

A logistic function (sigmoid) was chosen as the activation function for this implementation. This function is bandlimited, differentiable, positive everywhere and has a high value at its center. The design network for our compass heading modeling is a feed forward multilayer perceptron (MLP) network with three layers. To train the network, a back-propagation (BP) learning algorithm was applied. This is a simple algorithm for performance optimization through a supervised learning method (Principe *et al*., 2000). A BP learning algorithm can adjust connection weights to achieve the required network output. The inputs to the network, as shown in Figure 1, include raw compass output, total magnetic flux density, |H|, gyro heading, the vertical component of the gravity vector (or magnetic dip) and temperature.

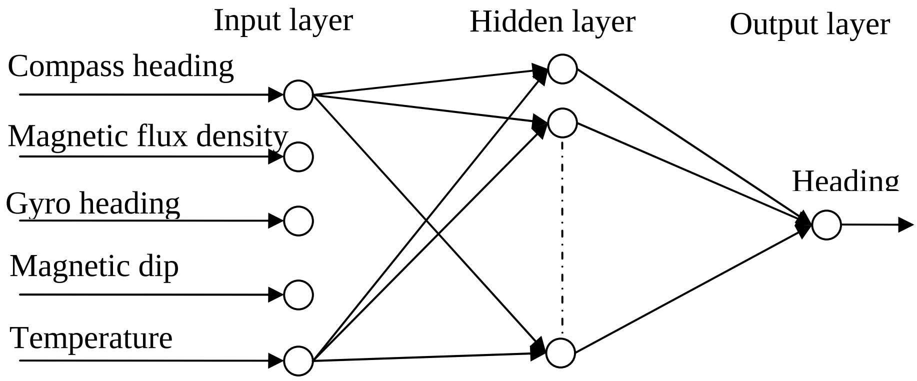


Figure 1. Neural network input and output parameters

The purpose of magnetometer and gyro integration at this level is not to determine the heading from the gyro, but to rectify the compass error. Magnetic dip, as a part of the inclination angle, represents the angle between the magnetic field vector and the local horizontal plane. This parameter is obtained from the product of the acceleration output, $\vec{f}^{\,b}$, and the magnetic field components, $\vec{H}^b$ (Hu *et al.*, 2005). As can be seen, in the dynamic calibration ANN, there is no need to determine magnetic field characteristics, such as bias, scale factor or declination angle. The trained ANN model automatically adjusts its connection weights to map the non-linear relation between true heading and uncalibrated magnetometer compass heading.

4. TEST RESULTS

To study the performance of the calibration methods (autocalibration and ANN approach), a test was undertaken at the Ohio State University campus, Columbus, Ohio, in May 2007. The HMR3000 was placed on the roof of a van, strapped to a wooden boom that extended outward from the body. A navigation grade LN100 INS and a dual frequency Novatel GPS receiver were installed in the OSU Center for Mapping GPSVan^{TM}. The conventional double difference carrier phase GPS/LN100 integration provided the reference solution in terms of position and attitude. The reference solution has a quality better than 2 cm in position and 10 arcsec in attitude.

A parking lot on the campus was chosen to collect the data. First, the van was driven relatively fast (<15 m/sec) along a rectangular shape trajectory. Then, it was moving slowly (<5 m/sec) in a circular trajectory several times. Figure 2 shows both trajectories, each with a different color. In this study, the second trajectory (black) was used for compass autocalibration and the first trajectory (red) was used for comparison checking of the autocalibration results. The output of the HMR3000 magnetometer compass shows quite a large error range of about (12±30)°, as shown in Figure 3.

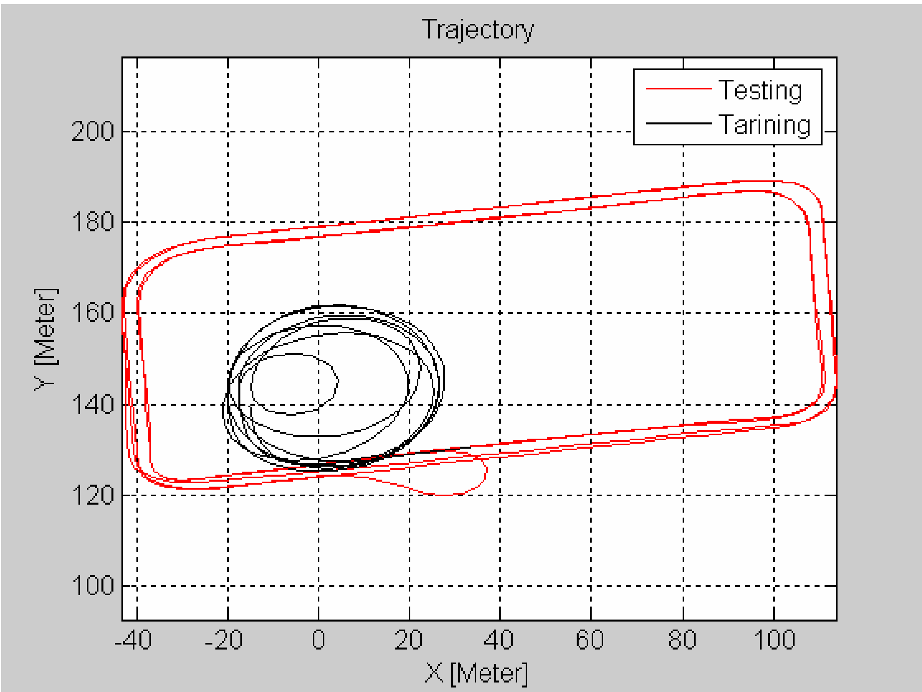


Figure 2: First trajectory (red: checking), second trajectory (black: training)

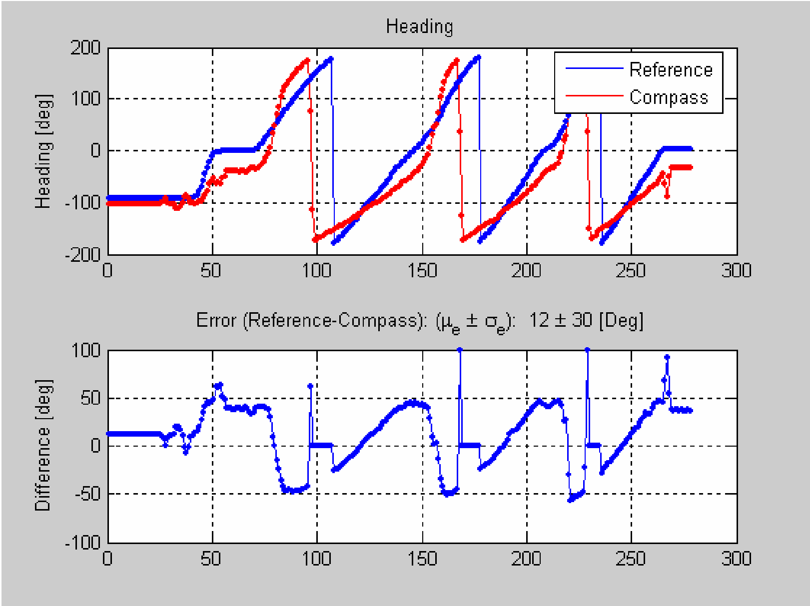


Figure 3. The difference between the reference heading and the compass heading before calibration

Figure 4 shows the roll and pitch angles output of the inclinometer, embedded in the HMR3000, with respect to the reference solutions. The uncertainties of about (0.04±0.9)° and (-0.3±0.7)° were observed for roll and pitch angles, respectively. Because of the low range of the tilt-x and tilt-y angles in this experiment, it is difficult to justify the reliability of the inclinometer sensor. More tests on sloping terrain are required.

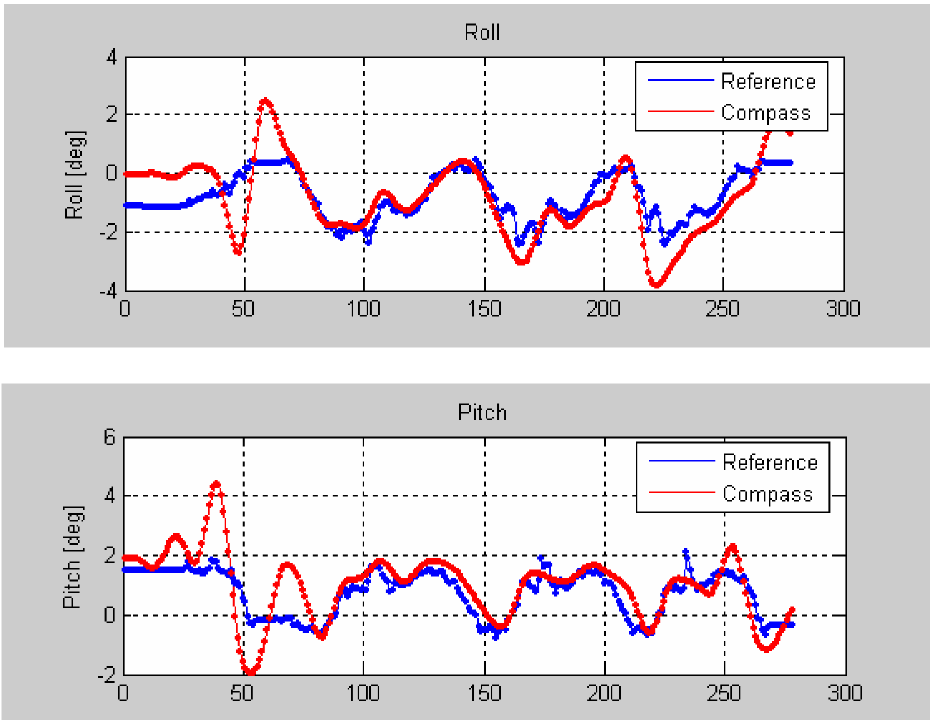
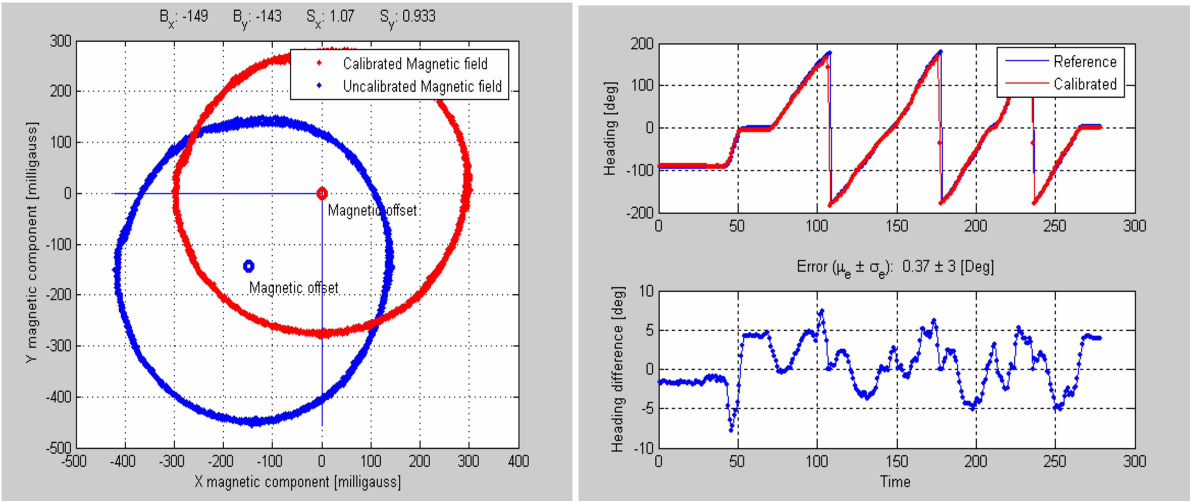


Figure 4. The difference between the references roll and pitch angles and inclinometer measured values

For the autocalibration procedure, the first step is to plot the horizontal components of the magnetic field. This plot should display a circle with the center at the origin in Figure 5 (a), if the errors are compensated. The subsequent processing includes the following tasks: (1) rotating the magnetic components to the horizontal plane using Eq.4, (2) estimating biases and scale factors using Eq.7, (3) applying the computed values to rectify the magnetic measurements using Eq.6, (4) computing the heading using Eq.1, and finally, (5) adding the declination angle to the compass output. Figure 5 (left and right panels) shows the calibrated magnetic field and the final results after applying the calibration parameters.



(a) (b)

Figure 5. (a) Interference of magnetometer reading for 360º rotation in level plane; (b) Heading error: calibrated compass heading with respect to the true heading

An error of about (0.4±3)º was obtained after the calibration the HMR3000 magnetic compass in this outdoor test, where the sensor was mainly impacted by the ferrous materials of the platform. Repeating the same procedure for the first dataset (which was not meant for autocalibration) results in an error range of (4.1±2.1)º; the main deviations between compass and reference headings were observed at corners. Upon returning to the straight path, the deviation resumed its expected range. As can be seen, the autocalibration of the magnetic compass improves the accuracy to better than 4-6º in this experiment.

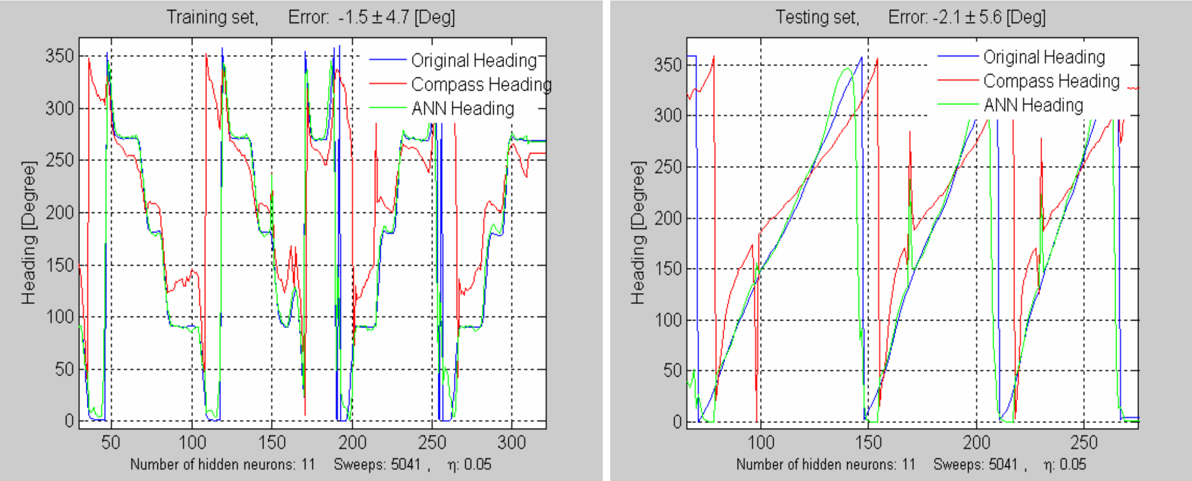
5. ANN COMPASS HEADING MODELING

The data used for the autocalibration procedure was selected to assess the performance of the ANN approach. As shown in Figure 1, five parameters were chosen as the input parameters, and 10-15 neurons were implemented in the hidden layer. Since each input parameter is of a different physical nature, they are normalized to the same numerical range with the mean removed. The correlations between the input parameters shown in Table 1 indicate that a sub-set of these parameters may be sufficient to model the heading. This is currently under investigation.

	Compass Heading	**|H|**	**Gyro Heading**	**Magnetic Dip**	**Temperature**
Compass Heading	1	0.41	0.36	0.39	0
|H|	0.41	1	0.73	0.61	0
Gyro Heading	0.36	0.73	1	0.52	0
Magnetic Dip	0.39	0.61	0.52	1	0
Temperature	0	0	0	0	1

Table 1. Correlation of input parameters.

The network was trained using the true heading obtained from the GPS/INS solution. It took about 5100 epochs to meet the 0.05 MSE error threshold. Figure 6 (left and right panels), shows the ANN heading compared to the reference heading for the training and testing dataset. The second dataset was used for training because it was longer and more dynamic (see Figure 2). An accuracy of (-1.5±4.7)º for training datasets and (-2.1±5.6)º for the testing dataset was achieved by compensating for the compass error.



(a) (b)

Figure 6. ANN method and compass heading calibration; (a) Training dataset; (b) testing dataset

6. CONCLUSION AND FUTURE WORKS

This paper reports on preliminary experiences gained by the autocalibration and dynamic calibration of a 3-axis HMR3000 magnetometer. The autocalibration procedure is based on using the constraint of a circular pattern, which is strong enough to display the magnetic field anomalies in terms of a bias and a scale factor for each axis. For dynamic calibration, a multilayer perceptron network, with one hidden layer and 5 input parameters, was proposed and implemented. In this method, the compass deviation, i.e. the difference between true heading and magnetometer compass heading, was used to train the network. Once the ANN is trained, it can provide calibrated heading based on the compass-measured heading and the additional input parameters. Both methods showed encouraging results, providing accuracy of about 4-6º heading performance. A more comprehensive performance analysis of the heading determination is currently under way for indoor experiments.

REFERENCES

Bourgeois B, Martinez A (1999) Magnetic Compass Dynamic Performance, Presented at the 11th International Symposium on Unmanned Untethered Submersible Technology, August 23-25, 1999, Durham, NH. NRL/PP/7442-99-0006.

Caruso MJ (1997) Application of Magnetoresistive Sensors in Navigation Systems, *Sensors and Actuators1997, SAE SP-1220*, February 1997, 15-21.

Caruso MJ (2000) Application of magnetic sensors for low cost compass system, Position Location and Navigation Symposium, IEEE PLANS, March 2000, 177-184.

Gebre-Egziabher D, Elkaim GH, Powell JD, Parkinson BW (2001) A non-linear, two step estimation algorithm for calibrating solid-state Strapdown magnetometers, 8th International St. Petersburg Conference on Navigation Systems, St. Petersburg, Russia. IEEE/AIAA.

Gouws DJ, Merwe EVD (2000) The implementation of a magnetic sensor on a dynamic platform. http://www.hmo.ac.za/old_site/magnetometer_on_dynamic_platform.pdf.

Grejner-Brzezinska DA, Toth C, Moafipoor S (2007) Pedestrian Tracking and Navigation Using Adaptive Knowledge System Based on Neural Networks and Fuzzy Logic, Journal of Applied Geodesy, in press (invited).

Haupt GT, Kasdin NJ, Keiser GM, Parkinson BW (1996) Optimal Recursive Iterative Algorithm for Discrete Nonlinear Least-Squares Problem. *AIAA Journal of Guidance Control and Navigation*, 19(3), 643 – 649.

Hoff B, Azuma R (2000) Autocalibration of an Electronic Compass in an Outdoor Augmented Reality System, Proceedings of IEEE and ACM International Symposium on Augmented Reality, Munich, Germany, 5-6 October 2000, 159-164.

Hu X, Liu Y, Wang Y, Hu Y, Yan D (2005) Autocalibration of an Electronic Compass for Augmented Reality, ISMAR 2005, 182-183.

Jekeli, C (2000) *Inertial Navigation Systems with Geodetic Applications*. Walter de Gruyter, New York, NY, USA, 2000.

Kao WW, Tsai CL (2006) Adaptive and Learning calibration of magnetic Compass, Measurement Science and Technology. Vol. 17, 3073-3082

Ladetto Q, Marminod B (2002) Digital magnetic compass and gyroscope integration for pedestrian navigation. *9th Saint Petersburg International Conference on Integrated Navigation Systems*, Saint Petersburg, Russia, May 2002.

Liu SW, Zhang ZN, Hung J (1989) A High Accuracy Magnetic Heading System Composed of Fluxgate Magnetometer and a Microcomputer, Records of NAECON' 89, *Proceedings of IEEE* (1989), 148–152.

Moafipoor S, Grejner-Brzezinska DA, Toth C (2007) A Fuzzy Dead Reckoning Algorithm for a Personal Navigator, *Proceedings of ION GNSS*, Fort Worth, Texas, September 25-28, 2007, CD ROM.

Principe JC, Euliano NR, Lefebvre WC (2000), Neural and Adaptive Systems, 1st edition, Wiley & Sons.

Schaffrin B, Felus Y (2005) On Total Least-Squares Adjustment with Constraints, International Association of Geodesy Symposia, Vol. 128, 417-421, Springer Berlin Heidelberg.

Tsai CL (2006) Motion-based Differential Carrier Phase Positioning, Heading Determination and its Applications. PhD dissertation, National Taiwan University of Science and Technology.

Wang JH, Gao Y (2006) A new magnetic compass calibration algorithm using neural networks. Measurement Science and Technology, Vol. 17, 153-160.

Wang L, Zhao Z, Shao YM, He HO (2006) A method of compensating magnetic compass's temperature error using BP network, *Journal of Physics:* Conference Series 48, 164-168.